DO NOW:

On your graphing calculator, graph, then sketch:

$x(t)=t - \sin t$

$y(t)=2 - 2\cos t$

for $0 \le t \le 10$

TSTEP$=\pi/24$

Use ZOOMFIT in ZOOM menu to have the calculator find the X and Y windows

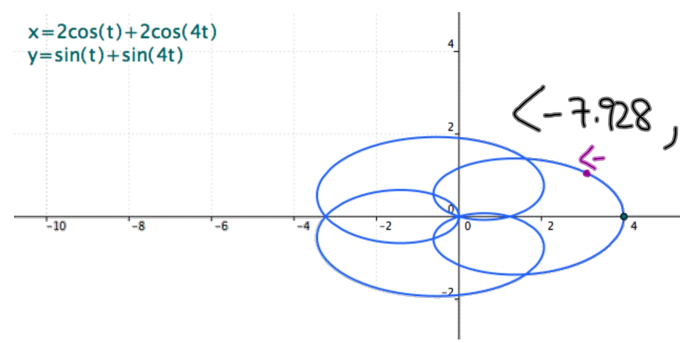
Use CALC and TRACE to explore what you can find out about the curve using the calculator

Use CALC and TRACE to explore

$x(t)=2\cos t + 2\cos 4t$

$y(t)=\sin t + \sin 4t$

$0 \le T \le 2\pi$



$\langle -7.928, -1.134 \rangle$

$\langle -, + \rangle$

In general, we will think of parametric equations as describing the motion (position) of a particle, with the variable t representing time

$$\langle x(t), y(t) \rangle$$

Some books, particularly the one that I copied problems, from will use physics style notation for position, we will use $\vec{i}$ to represent a unit vector in the x-direction and $\vec{j}$ to represent a unit vector in the y-direction, ie:

$$\vec{i} = (1,0) = \hat{i} \qquad \vec{i}$$

$$\vec{j} = (0,1)$$

$X(\pi) = 2$
$y(\pi) = -3$

The position of a particle at time t is given by:
$r(t)=x(t)\,\vec{i} + y(t)\,\vec{j}$ $\quad r(t) = 2(1,0) - 3(0,1) = (2,0) - (0,3) = (2,-3)$

Position Velocity Acceleration Speed April 25, 2017

There are three main terms that we need to know when we think about a particle whose motion is described parametrically:

VELOCITY ACCELERATION SPEED

$\langle 2, -3 \rangle$

Remember, velocity and acceleration are vectors
Speed is a scalar = magnitude of velocity

$2(1,0) - 3(0,1) = \langle 2, -3 \rangle$

To calculate velocity and acceleration, we differentiate term by term, that is

if

$$r(t) = x(t)\vec{i} + y(t)\vec{j} \qquad \langle x(t), y(t) \rangle$$

then

$$v(t) = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} \qquad \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

and

$$a(t) = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} \qquad \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle$$

Since speed is the magnitude of velocity, we calculate it by taking the square root of the sum of the squares of the components of the velocity:

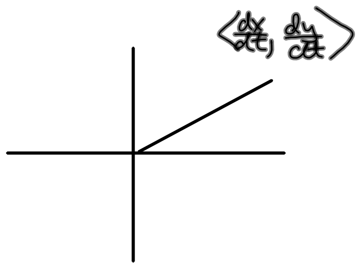
If $r(t) = x(t)\hat{i} + y(t)\hat{j}$

so $v(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

then

speed= $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

length of the velocity vector



Let's work an example together:
#24 from Varberg-Purcell page 653

$$r(t) = \left(3t^2 - 1\right)\vec{i} + t\,\vec{j}$$

Find v(t) and a(t) and the speed.

$v(t) = 6t\hat{i} + \hat{j}$

$a(t) = 6\hat{i}$

Speed $= \sqrt{(6t)^2 + 1^2} = \sqrt{36t^2 + 1}$

Evaluate these at t=1/2.

$v\left(\frac{1}{2}\right) = 3\hat{i} + \hat{j}$ Speed$\left(\frac{1}{2}\right) = \sqrt{10}$

$a\left(\frac{1}{2}\right) = 6\hat{i}$

Practice Problems: Packet, page 3, #23-33 (odd)

$$\frac{dx}{dt} = \frac{3\ln(\cos t^2)}{\arctan\left(\frac{1}{t}\right)}$$

$$\frac{dy}{dt} = \frac{5\sec\left(\frac{1}{t^2}\right)}{t^4+3}$$

What is the acceleration vector at $t=3$?

Use NDERIV!