AP Calculus

Arc Length
Parametric/Vector Equations

Mass Insight EDUCATION MASS MATH + SCIENCE INITIATIVE

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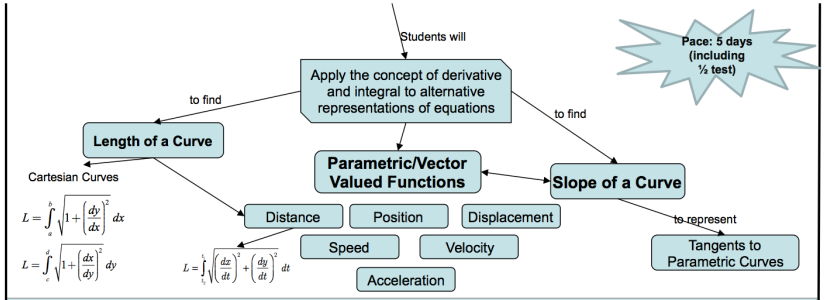
Do Now: Take out your graphing calculator, put it in parametric mode

HW: DUE THURSDAY:Purple Parametrics Packet: P.1 #66 (matching) and #11-25 odd (use calculator, SKETCH the graph on paper, don't need to eliminate the variable)

Unit 11: Ostebee references (4.9, 13.1, 13.2)

Last Unit	**The Calculus of Parametric and Vector Equations**	*Next Unit*
Differential Equations		Review for the AP Exam



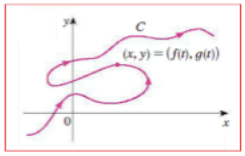
Required Skills/Vocabulary: principles of differential calculus, the definite integral, properties of the definite integral, antiderivative, parametric curves and their derivatives, integral as accumulator, Riemann Sums, vector equations and their derivatives, velocity and acceleration vectors, unit circle, equation of a circle, trigonometric equivalents of Cartesian coordinates.

Unit Self Test Questions

- Be able to do /explain the following processes:
  - Find the length of an arc/curve (in Cartesian, parametric/vector form),
  - Find the velocity and acceleration vectors and all measurements related to 2-dimensional motion given parametric/vector equations, (position, speed, distance travelled…),
  - The Cartesian equivalents of vector/polar coordinates/equations, **AND**
  - Solve initial value problems and represent solutions in 2-dimensions.

Intro: Parametric Equations

Imagine a particle moving in the plane along a curve *C* during a time interval. In the figure to the right, we can not model this curve using *y = f(x)* because *C* fails the vertical line test. We can, however, let time be the independent variable and model the *x*- and *y*-coordinates as functions of time.



Parametric Equations

We can represent parametric equations in several ways. The important idea is that the independent variable is called the parameter and the x- and y-coordinates are functions of the parameter. Often, but not always, the parameter is time *t*.

$$P : t \rightarrow \begin{cases} x = f(t) \\ y = g(t) \end{cases} \qquad P(t) = \Big(f(t), g(t)\Big)$$

Example

- Consider the curve given by the parametric equations which concerns a particle moving in the plane during a six second time interval

$$P : t \to \begin{cases} x = t^2 - 2t \\ y = t + 1 \end{cases} \text{ for } -2 \le t \le 4$$

What are the precalculus questions that are worth asking?

Parametrics.ggb

① Starting and ending point?
Intercepts? Crossing points? Repeated sections?
② When is it going up or down.
When is it moving left or right?
Is there any time it "stops moving"?
③ Can we rewrite parametric as $y=f(x)$ or $x=g(y)$.

Graphing Parametrics

In MODE select PAR

$\begin{cases} x_1(t) \\ y_1(t) \end{cases}$

Variable t is on $\frac{X,t}{\theta,n}$ button

WINDOW

Tmin

Tmax

Tstep = ~~.5~~ ~~.01~~ 0.1

Can use ZOOM FIT to have calc find X and Y limits

[2nd] [ON] - Stops whatever calc is doing

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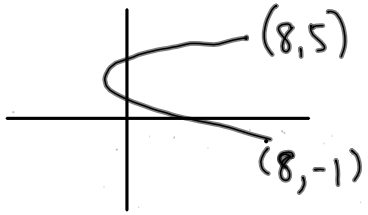
BC Calculus: Unit 1 Parametric/Vector Equations Names

1. Consider the parametric curve:

$$P(t)=\begin{cases} x=t^2-2t \\ y=t+1 \end{cases} \text{ for } t\in[-2,4]$$

$$V(t)=\begin{cases} \frac{dx}{dt}=2t-2 \\ \frac{dy}{dt}=1 \end{cases}$$

Sketch the curve in the space below and then answer the questions that follow (If you know/remember the Calculus of how to answer these questions...GREAT. Use it. If not, do the best to interpret what you can from the graph and equations and we will review these ideas):



a) What are the initial and terminal points of the curve?

b) Consider a particle who path is described by the parametric curve. When is the particle moving to the left? To the right?

Left from t=-2, until t=1, then moves right

c) When does the particle change direction?

At time t=1, Where? (-1,2)

d) Does the particle ever stop moving? When?

No, both directions don't stop at the same time

e) Is the particle ever moving vertically (strictly north/south)? When?

At t=1

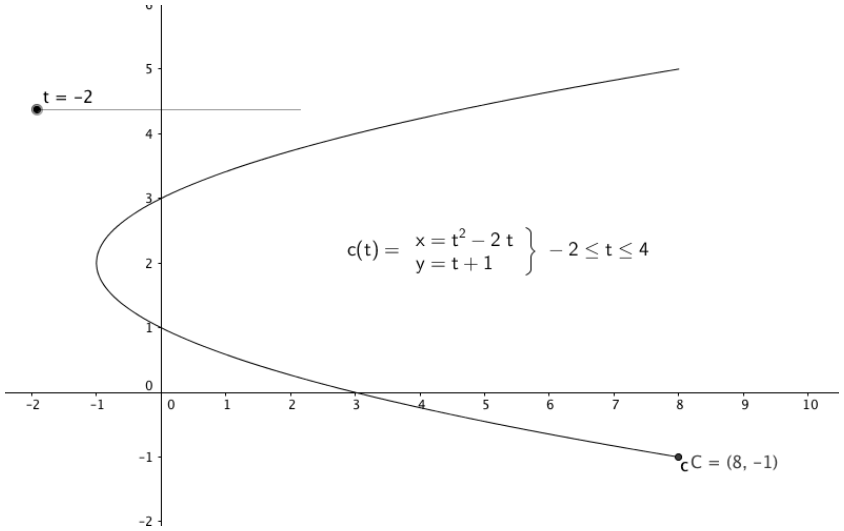
f) Is the particle ever moving horizontally (strictly east/west)? When?

NO.

g) When is the particle moving faster/slower?

As x get bigger, speed increases.

h) Express the parametric equation as a Cartesian equation, ~~y(x)~~ x=f(y)



My questions

- With a table of values, answer these questions:
  - What is the starting point when t = -2?
  - What is the ending point when t = 4?
  - At what times t does the particle pass through the coordinate axes?
- With a graph, answer these questions:
  - At what times t is the particle moving to the left? to the right?
  - At what times t is the particle moving upward? downward?
  - At what time t is the particle moving vertically?
- Other questions:
  - What is an equation of the graph as a function in one variable?
  - How would we adjust the parametric equations to have the particle retrace its steps?

Example- Creating a table of paired values

$$P: t \to \begin{cases} x = t^2 - 2t \\ y = t + 1 \end{cases} \text{ for } -2 \le t \le 4$$

or the position of the particle is given by

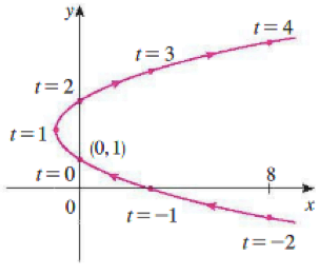
$$P = \left(t^2 - 2t, t + 1\right)$$

time	-2	-1	0	1	2	3	4
Position	(8,-1)	(3,0)	(0,1)	(-1,2)	(0,3)	(3,4)	(8,5)

Example- Creating a graph with dynamics

$$P: t \to \begin{cases} x = t^2 - 2t \\ y = t + 1 \end{cases} \text{ for } -2 \le t \le 4$$

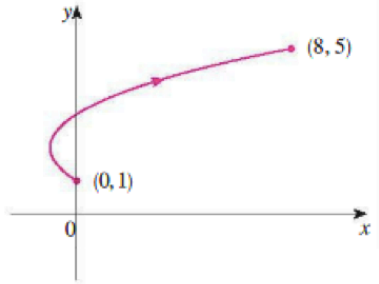
t	x	y
−2	8	−1
−1	3	0
0	0	1
1	−1	2
2	0	3
3	3	4
4	8	5



Example- Changing the time interval

$$P: t \to \begin{cases} x = t^2 - 2t \\ y = t + 1 \end{cases} \quad \text{for } 0 \le t \le 4$$

time	x	y
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5



Parametrics Removing the parameter

$x = t^2 - 2t$

$y = t + 1$

It follows that $t = (y - 1)$. Substitution gives

$x = (y-1)^2 - 2(y-1) = y^2 - 2y + 1 - 2y + 2$

$\therefore x = y^2 - 4y + 3$

or $x = (y-3)(y-1)$