

- 1.) For the differential equation $\frac{dy}{dt}$ #1, suppose the initial condition is $y(0) = 10$. How long until $y = 100$?

$$\begin{aligned}\frac{dy}{dt} &= 0.5y \\ \int \frac{dy}{y} &= \int 0.5 dt \\ \ln|y| &= \frac{1}{2}t + \ln 10 \\ \ln y &= \frac{1}{2}t + C\end{aligned}$$

$$\begin{aligned}e^{\ln|y|} &= e^{\frac{1}{2}t + \ln 10} \\ |y| &= 10e^{\frac{1}{2}t} \\ y &= \pm 10e^{\frac{1}{2}t} \\ y &= 10e^{\frac{1}{2}t} \\ 100 &= 10e^{\frac{1}{2}t} \\ 10 &= e^{\frac{1}{2}t} \\ 2 \ln 10 &= t \\ 4.605 &= t\end{aligned}$$

- 2.) For the differential equation $\frac{dy}{dt}$ #2, find $\frac{d^2y}{dt^2}$ in terms of y . What does $\frac{d^2y}{dt^2}$ tell you about the solution curves for $y > 0$?

$$\begin{aligned}\frac{dy}{dt} &= -0.25y \\ \frac{dy}{dt} &\propto -0.25y \\ \frac{d^2y}{dt^2} &= -0.25 \cancel{\frac{dy}{dt}} \\ &= -0.25(-0.25)y > 0\end{aligned}$$

all solution curves $y = f(t)$
must be concave ~~up~~ up

- 3.) For the differential equation $\frac{dy}{dt}$ #3, use separation of variables to find the particular solution $y = y(t)$ with the initial condition $y(0) = 2$.

$$\frac{dy}{dt} = -0.15(y-5)$$

$$\int \frac{dy}{y-5} = \int -0.15 dt$$

$$\ln|y-5| = -0.15t + C$$

$$|y-5| = e^{-0.15t+C}$$

$$y-5 = \pm e^C \cdot e^{-0.15t}$$

$$y = \boxed{\pm e^C} e^{-0.15t} + 5$$

$$2 = Ae^0 + 5$$

$$-3 = A$$

$$\boxed{y = -3e^{-0.15t} + 5}$$

4.) For the differential equation $\frac{dy}{dt}$ #4, compute $\frac{d^2y}{dt^2}$ in terms of y and use this justify

why the solution curves are always concave down for $y > 0$:

$$\frac{dy}{dt} = \frac{2}{y}$$

$$\frac{d^2y}{dt^2} = -\frac{2}{y^2} \cdot \frac{dy}{dt} = -\frac{4}{y^3} < 0$$

$$y > 0 \Rightarrow -\frac{2}{y^2} < 0$$

since $\frac{d^2y}{dt^2} < 0$
for $y > 0$

∴ The curves
 $y = f(t)$ are
always concave
down

5.) For the differential equation $\frac{dy}{dt}$ #5, suppose that a particular solution $y = y(t)$ passes

through the point $(1, 2)$. Use the line tangent to the graph of $y = y(t)$ at $t = 1$ to estimate $y(1.1)$.

$$\frac{dy}{dt} = y(t-5)$$

$$y-2 = 8(x-1)$$

$$L(x) = y = -8(x-1) + 2$$

$$y(1.1) \doteq L(1.1) = -8(+1.1-1) + 2 \\ = -1 + 2$$

Tangent

pt $(1, 2)$

$$\text{slope} = \left. \frac{dy}{dt} \right|_{t=1} = 2(1-5) - 8$$

$$y(1.1) \doteq L(1.1) = 1.2$$

6.) For the differential equation $\frac{dy}{dt}$ #6, algebraically show that the function $y(t) = \frac{4}{1+e^{-t}}$

satisfies the differential equation.

$$\frac{dy}{dt} = 0.25y(4-y)$$

$$\frac{dy}{dt} = y - \frac{y^2}{4}$$

$$\frac{dy}{dt} = y \left(1 - \frac{y}{4}\right)$$

$$\frac{dy}{dt} = \left(\frac{4}{1+e^{-t}}\right) \left(1 - \frac{\frac{4}{1+e^{-t}}}{4}\right)$$

$$\frac{dy}{dt} = \frac{4}{1+e^{-t}} \left(\frac{1+e^{-t}}{1+e^{-t}} - \frac{1}{1+e^{-t}}\right)$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{4}{1+e^{-t}} \left(\frac{1+e^{-t}-1}{1+e^{-t}}\right) \\ &= \frac{dy}{dt} = \frac{4e^{-t}}{4(1+e^{-t})^2} \\ &= 4(-1)(1+e^{-t})(-e^{-t}) \end{aligned}$$

$$\frac{4e^{-t}}{(1+e^{-t})^2} = \frac{4e^{-t}}{(1+e^{-t})^2}$$

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