HW: Page 640 #1, 3, 6, 11 (Euler's method)

DO NOW:

Questions from HW?

1) Copy the diff eq for yesterday's lab onto your sheet.
2) In your group, do problem #11 (slope field matching) on page 5 of your Diff Eq packet.

If dy/dt=y(t-5) and y(1)=2; find the equation of the tangent to the graph of y(t) at t=1. Use the tangent line to approximate y(1.1).

$$\left.\frac{dy}{dt}\right|_{(1,2)} = 2(1-5) = -8$$

$$y_1(t) = 2 + -8(t-1)$$

$$y_1(1.1) = 2 + -8(1.1-1)$$

$$= 1.2$$

$$\approx y(1.1)$$

e

We will look at Euler's (oiler's) method today. Euler's method takes the idea of "following the tangents" and makes it numerical.

We will work with the diff eq dy/dt=y(t-5) and y(1)=2 and look at the interval [1,2]
Let's figure out how to fill in this table.

n	t	y	$\frac{dy}{dt}$
0	1	2	$\frac{dy}{dt}=2(1-5)=-8=m_0$
1	1.1	$y_1(1.1)=1.2$	$\frac{dy}{dt}=1.2(1.1-5)=-4.68$ m_1
2	1.2	.732	

$y_1(t)=-8(t-1)+2$

$y_2(t)=-4.68(t-1.1)+1.2$

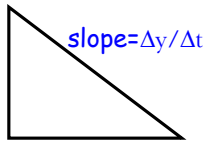
$=.732$

How do we do Euler's Method by hand?

Given $y'=F(t,y)$ and $y(t_0)=y_0$

Let Δt be the length of the time step

We get a table:



n	t	y	m
0	t_0	y_0	$F(t_0,y_0)=m_0$
1	$t_1=t_0+\Delta t$	$y_1=y_0+\Delta y$ $\Delta y=m_0\cdot\Delta t$	$F(t_1,y_1)=m_1$
2	$t_2=t_1+\Delta t$	$y_2=y_1+\Delta y$ $\Delta y=m_1\cdot\Delta t$	

$\Delta y=m\cdot\Delta t$

http://tube.geogebra.org/student/m144927

Problem #6 from yesterday's lab is logistic growth; the general form is

$$P' = kP(C-P)$$

In problem #6, k = 0.25 and C = 4 $\quad P' = 0.25P(4-P)$

Let's start at (0, 1) and do two steps by hand with $\Delta t = 0.5$, this will approximate P(1)

We can then see what we get on the computer if we decrease Δt

n	t	P	P'
0	0	1	$P' = .75$
1	$t_1 = 0 + \frac{1}{2} = 0.5$	$y_1 = 1 + 0.5(0.75) = 1.375$	$P' = .9023$
2	$t_2 = 0.5 + .5 = 1$	$y_2 = 1.375 + .5(.9023) = 1.826$	

DIDN'T DO IN CLASS TODAY!!

What are the properties of logistic growth?

a) What is the limit as $t \to \infty$? Does it matter whether we start below or above carrying capacity?

Carrying Capacity $\quad P\,640\ 1,37,11$

b) When is the population growing most rapidly? (Where is the maximum for P'?)

Let's look at a graph of P vs. P':

$P' = kP(C-P)$

Maximum of P' is when $P = \frac{C}{2}$, ie half of carrying capacity