Name: Date:
504: BC Calculus (AP) Slope Field Clinic

What is a slope field?

A slope field or direction field for the first order differential equation:

$$\frac{dy}{dx} = f(x, y)$$

is a plot of short line segments with slopes $f(x, y)$ for a lattice of points in the (x, y) plane.

In other words… a slope field shows you pieces of tangents to a curve at any given point in the coordinate plane. A slope field is a road map of the derivative which gives you directions for how to sketch your antiderivative. It is a sketch of the *differential equation* before you solve. When you sketch the slope field it gives you a visual of the family of antiderivatives. *Slope fields are particularly useful for solving differential equations for which we cannot separate variables in order to integrate.*

Goals of this Clinic:

1) For you to be able to sketch your own slope field by hand
2) For you to be able to match a slope field with a given differential equation
3) For you to be able to use a slope field and initial condition to sketch a specific solution to an initial value problem.

By the end of class today, you want to feel as though you have reached Goals 1 and 2 of the Slope Field Clinic. After our next class we will have reached Goal 3.

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1) Sketch the slope field by hand for the following differential equation:

$$\frac{dy}{dx} = x - y$$

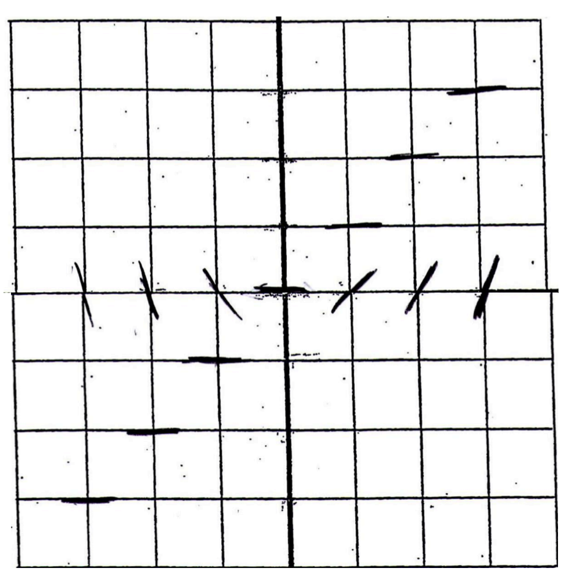
2) Make a table:

(x, y)	Work	$\frac{dy}{dx}$
$(0,0)$	0-0	0
$(1,0)$	1-0	1
$(2,0)$	2-0	2
$(3,0)$	3-0	3
$(-1,0)$	-1-0	-1
$(-2,0)$	-2-0	-2
$(-3,0)$	-3-0	-3

This 3rd column represents the slope of the tangent lines at the ordered pairs from the 1st column.

You do not need to do these calculations for EVERY ordered pair. Look for patterns. For Example, look at the values of x and y which will make $\frac{dy}{dx}$ zero. In this case, whenever $x = y$, $\frac{dy}{dx} = 0$. What does this mean? It means that the tangents will be horizontal along the line $y = x$.

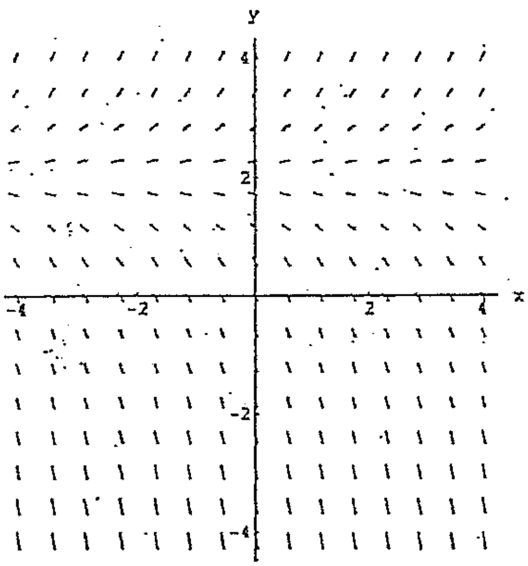
3) The graph below shows the start of the sketch for the slope field for $\frac{dy}{dx} = x - y$ based on the information discussed above. **Finish sketching this slope field.**



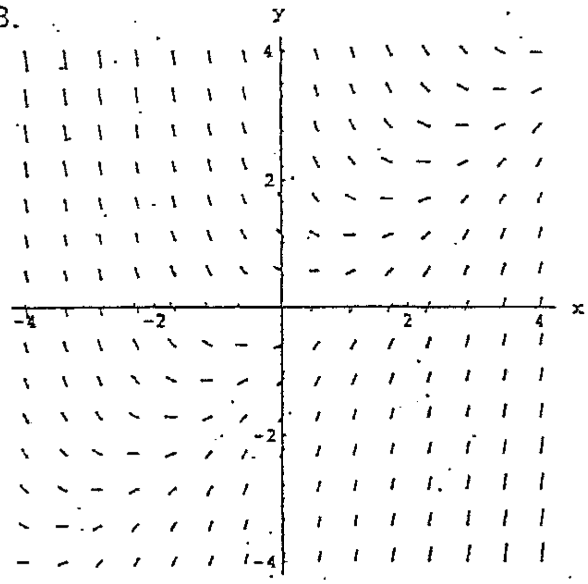
2. Match the slope field with its differential equation. Explain the reasons for your choices.

a. $y' = y - 2$ b. $y' = x - y$ c. $y' = x^2 - y^2$ d. $y' = x^3 - y^3$

A.



B.



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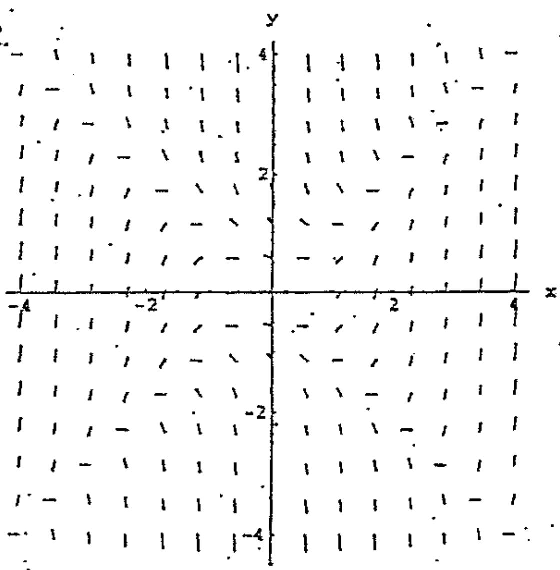
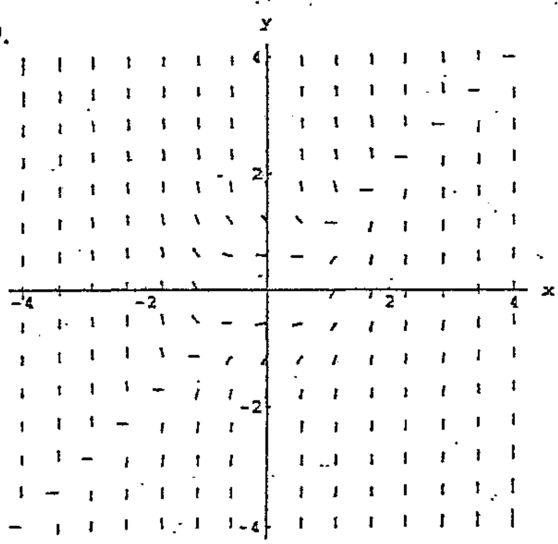
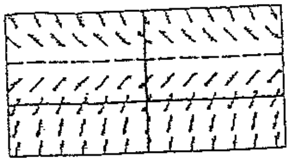


D.

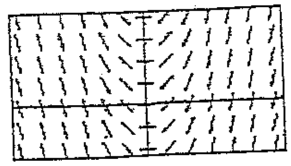


Match the slope fields with their differential equations.

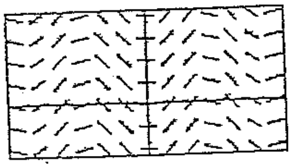
(A)



(B)



(C)



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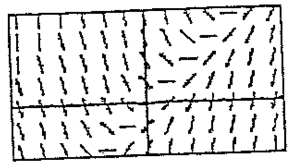
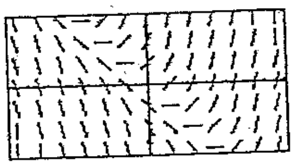


7. $\frac{dy}{dx} = \sin x$ 8. $\frac{dy}{dx} = x - y$ 9. $\frac{dy}{dx} = 2 - y$ 10. $\frac{dy}{dx} = x$

Match the slope fields with their differential equations.

(A)



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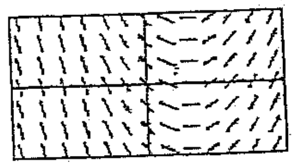

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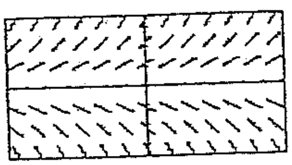

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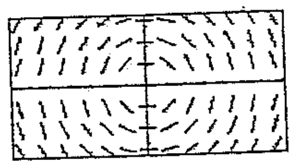

11. $\frac{dy}{dx} = .5x - 1$ 12. $\frac{dy}{dx} = .5y$ 13. $\frac{dy}{dx} = -\frac{x}{y}$ 14. $\frac{dy}{dx} = x + y$

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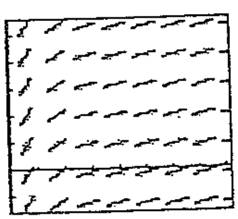
15. (From the AP Calculus Course Description)



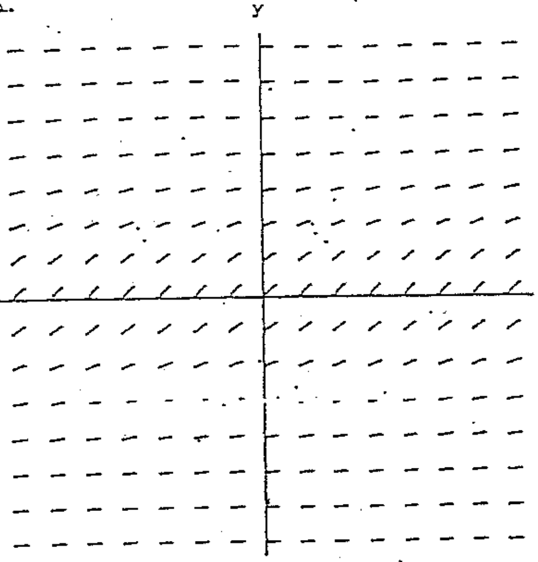
The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

1. Match the slope fields with their differential equations.

(a) $y' = y$ (b) $y' = -y$ (c) $y' = 1 + y^2$ (d) $y' = 1/y$ (e) $y' = 1/(1+y^2)$

A.



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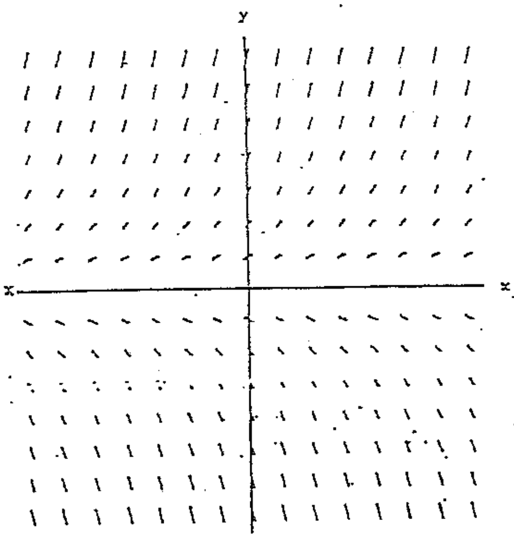

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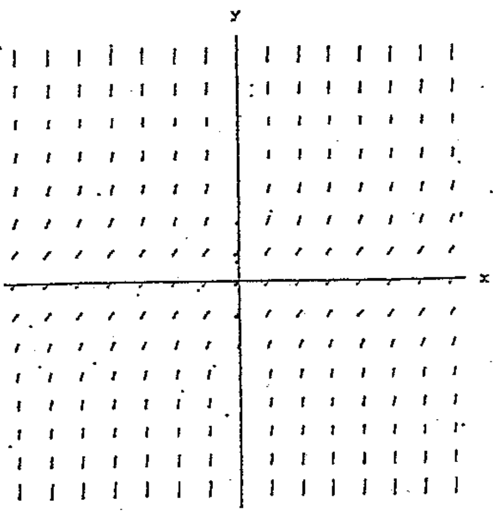
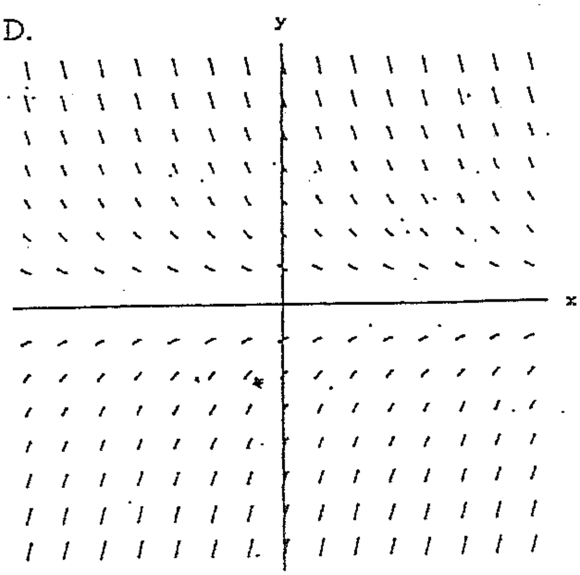
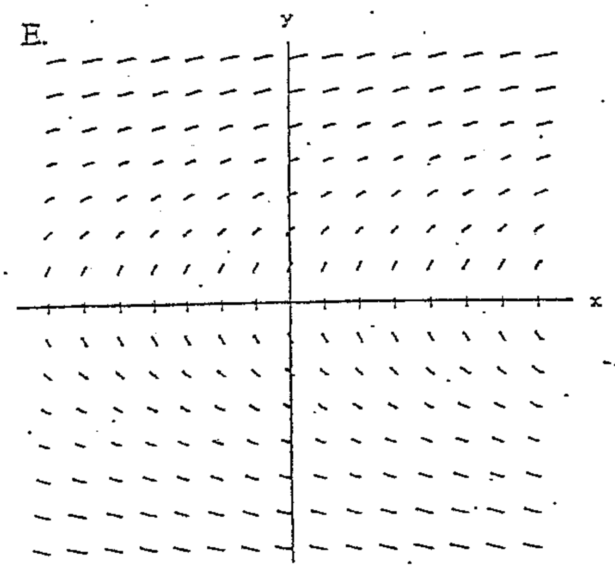


D.

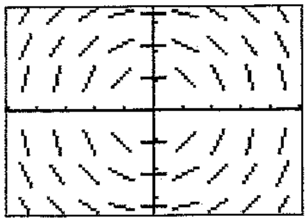


E.

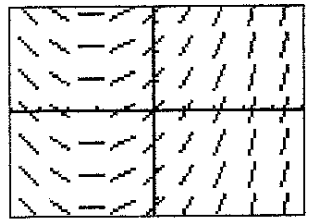


Match the slope fields with their differential equations.

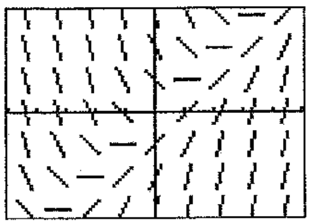
(A)



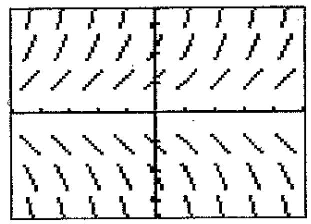
(B)



(C)



(D)



15. $\dfrac{dy}{dx} = \dfrac{1}{2}x + 1$

16. $\dfrac{dy}{dx} = y$

17. $\dfrac{dy}{dx} = x - y$

18. $\dfrac{dy}{dx} = -\dfrac{x}{y}$

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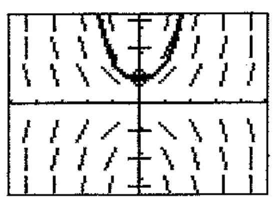
19. The calculator drawn slope field for the differential equation $\dfrac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
(a) Sketch the solution curve through the point (0, 2).
(b) Sketch the solution curve through the point $(0, -1)$.



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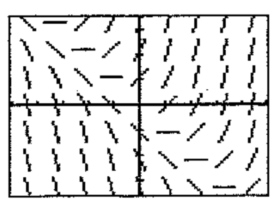
20. The calculator drawn slope field for the differential equation $\dfrac{dy}{dx} = x + y$ is shown in the figure below.
(a) Sketch the solution curve through the point (0, 1).
(b) Sketch the solution curve through the point $(-3, 0)$.



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