HW: Page 254 (Textbook) #1 - 11 (all)

DO NOW:

Find ONE solution to each of the following differential equations:

1) $f'(x)=6x+5$ $\quad f(x)=3x^2+5x+1$

2) $y'(t)=y(t)$ $\quad y(t)=e^t$

3) $g''(x)=-g(x)$ $\quad g(x)=\sin x \quad g(x)=\cos x$

How do we find the "family" of functions that is a solution for each of the diff eqs

1) $f(x)=3x^2+5x+C$

2) $y(t)=Ce^t \qquad e^{t+5}=e^5e^t$

3) $g(x)=A\sin x+B\cos x$

$$V(t)=5\cos t$$
$$P(t)=V(t)=5\sin t+C$$

One of the main ideas about differential equations is that the solution is a function (or family of functions) that satisfy the differential equation.
An Initial Value Problem (IVP) yields a unique solution.

Looking at
1) $f'(x)=6x+5$
2) $y'(t)=y(t)$

Factor: x^2-6x+5
Need to know how to multiply, in order to guess & check

For each one, find the solution that passes through (0, 42).

$f(x)=3x^2+5x+42$

$y(t)=42e^t$

Differential Equations are easy to check: if I give you what I think is the solution, you can determine if it is correct.

They are hard to solve; if you study Math or Engineering or Physics, you may need to take an entire course in Differential Equations.

Solving differential equations is similar to factoring polynomials; in most cases you cannot find an answer. However, using technology, we can now find out more about the solutions to many differential equations than we could in the past.

In this unit, we will learn how to solve some differential equations and learn how some of the technology is used to understand the solutions of differential equations.

Let's consider a simple real-world situation that is modeled by a fairly simple diff eq:
If I pour a hot cup of coffee into my mug and then walk around with it, the temperature of the coffee will decrease until it is the same temperature as the room I am in.

(T= ambient temperature)

The rate at which the coffee will cool is proportional to the difference between the current temperature of the coffee and the ambient temperature.

k = proportionality constant

If y= temperature of my coffee at time t, what is the differential equation it must satisfy?

$$\frac{dy}{dt} = k\left(y(t) - T\right)$$

$$y' = k(y - T)$$

This equation is known as Newton's law of cooling.

$y' = k(y-T)$

My claim is that the function $y(t) = T + Ae^{kt}$ is a solution

$$y(t) = T + Ae^{kt}$$

Let's show that this function satisfies the differential equation:

$$y' \stackrel{?}{=} k(y-T)$$

$$kAe^{kt} \stackrel{?}{=} k(T + Ae^{kt} - T)$$

$$kAe^{kt} = kAe^{kt}$$

What is the value for A? $y(0) = T + A$

$$A = y(0) - T$$

What happens to y(t) as $t \to \infty$? Does that agree with what we see happening in the real world?

If coffee is cooling, $k<0$

$$\lim_{t\to\infty} T + Ae^{kt} = T$$

Let's work a particular example together:
Suppose that room temperature is 20° C and our coffee starts at a temperature of 95° C and after 2 minutes it is at 70° C.
What is the function y(t) for the temperature after t minutes?
What happens to the temperature as $t \to \infty$

$$y(t) = T + Ae^{kt}$$

$$y(t) = 20 + Ae^{kt}$$

$$y(0) = 20 + Ae^{k\cdot 0}$$

$$95 = 20 + A \Rightarrow A = 75$$

$$y(t) = 20 + 75e^{kt}$$

$$70 = y(2) = 20 + 75e^{2k}$$

$$50 = 75e^{2k}$$

$$\frac{2}{3} = e^{2k}$$

$$\ln\left(\frac{2}{3}\right) = 2k$$

$$\frac{\ln\left(\frac{2}{3}\right)}{2} = k$$

$$k = -0.203$$

$$y(t) = 20 + 75e^{-0.203t}$$