HW: Page 108 #16 - 23, 31a,31b

Upcoming Tests: Friday, March 17, 2017 Series Test

THESE TWO TESTS WILL COUNT ON FOURTH QUARTER

Tues, March 21, 2017: AP Practice Free Response

Wed, March 22, 2017: AP Practice MC, Non- Calculator

DO NOW:

Any questions from HW for today?

26 $\cos(\frac{1}{n}) \to 1$

28 $n\cos(\frac{1}{n}) \to \infty$

25 $\sin(\frac{1}{n})$ limit comp (use L'Hopital) with $\frac{1}{n}$

$\boxed{\sin x \approx x} - \frac{x^3}{3!} + \frac{x^5}{5!}$

28 $n\sin(\frac{1}{n})$, use L'Hopital $\frac{\sin(\frac{1}{n})}{\frac{1}{n}}$

$\to \infty$ doesn't go to zero

C 29 $\frac{1}{n}\sin(\frac{1}{n})$, lim comp w $\frac{1}{n^2}$

24 $\sum \frac{n^2+2^n}{n!}$ looks like $\frac{2^n}{n!}$

log < polynomials < exponential < factorial

$\sum \frac{e^{1/n}}{n^2}$ $\int e^{x^{-1}} \cdot x^{-2}\,dx$

$\sum_{n=0}^{\infty} \frac{1}{an+b}$ $a>0$ diverges

Looks like $\frac{1}{n}$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{an+b}} = a > 0$

Recall that the syntax for creating a sequence on the TI-83 (or 84) is **seq (rule, variable, start index, final index)** and the command **seq** is found under LIST OPS (choice #5)

Create a sequence consisting of the first 500 terms of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}+\ldots$ and store that sequence in L_1

$\text{Seq}((-1)^{(x+1)}/x,\ x, 1, 500) \to L1$

Step=1

Use the cumulative sum command, found right below the sequence command, to store the cumulative sums for this sequence in L_2

(Syntax is **cumsum(sequence)**, you can use the name L_1 for the sequence)

What do you notice about the sequence of partial sums?

Appear to converge to .6921482

VARYING SIGN SERIES

We need to look at series where not all of the terms are the same sign in order to complete our study of power series and Taylor series.

We will begin with the simplest case of varying signs: the case where the terms strictly alternate. An example (an important one, it turns out) is $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots$. Such series are called "alternating series."

Alternating Series and the Alternating Series Test

Definition: An **alternating series** is a series in which the terms strictly alternate in sign. In other words, no two consecutive terms have the same sign.

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Practice 1

Which of the following series are alternating?

a. $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n!}$ b. $\sum_{n=0}^{\infty}(-3)^n$ c. $\sum_{n=0}^{\infty}\frac{\cos(n\pi)}{n^2+1}$ d. $\sum_{n=1}^{\infty}\sin(n)$

$-1+1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}$

$1-3+9-27+\ldots$

$1-\frac{1}{2}+\frac{1}{5}-\frac{1}{10}+\ldots$

$\sin 1+\sin 2+\sin 3+\sin 4$

$+ \quad + \quad + \quad -$

In order to gain an understanding of convergence for alternating series, let's look at the one mentioned above:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots+(-1)^{n+1}\cdot\frac{1}{n}+\cdots.$$

This is the alternating harmonic series. We can look at the individual (signed) terms, a_n, as well as the partial sums, s_n.

n	1	2	3	4	5	6	7	8	9	10
a_n	1	-0.5	0.3333	-0.25	0.2	-0.8333	0.1429	-0.125	0.1111	-0.1
s_n	1	0.5	0.8333	0.58333	0.7833	0.6167	0.7595	0.6345	0.7456	0.6456

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Do you see what's happening? The partial sums are bouncing up and down around some number, but all the while they are zeroing in on something. Maybe it will be clearer if we look a bit farther out.

n	100	101	102	103	104	105	106	107	108
a_n	-0.01	0.0099	-0.0098	0.0097	-0.0096	0.0095	-0.0094	0.0094	-0.0093
s_n	0.6882	0.6981	0.6883	0.6980	0.6884	0.6979	0.6885	0.6978	0.6885

https://www.geogebra.org/m/FBaEmDy7

Alternating Series March 07, 2017

Figure 8.1 is an attempt to make this clearer. The blue dots represent the terms a_n of the alternating harmonic series (sign and all) as a function of n. The red dots represent the corresponding partial sums s_n. (Note that $a_1 = s_1$, so only one dot is visible on the graph for n = 1.) As you can see, the blue dots are "funneling in" toward zero, confirming that the series passes the n^{th} term test. The partial sums, for their part, are tending toward the limit $s = 0.6931$. Look at the graph one n-value at a time. A blue dot below the n-axis corresponds to a red dot below the dotted line; the negative term has pulled the partial sum below its eventual limit. This is followed, though, by a blue dot above the n-axis; at the same time the partial sum dot has drifted back above 0.6931, but not as far as it had been.

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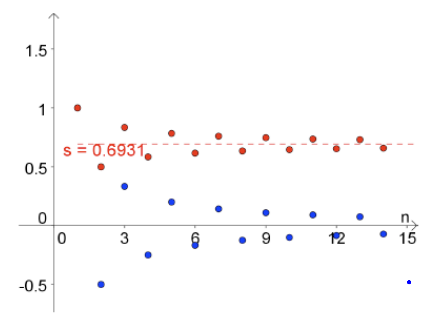

Figure 8.1: Terms and partial sums of the alternating harmonic series

The above discussion identified two key factors responsible for the alternating harmonic series converging. First and most fundamentally, the terms alternate in sign; this allows them to cancel partially. Second, the terms are decreasing in size so that the cancellation is never complete; this is why the oscillation in the partial sums is damped, leading to a well-defined limit. And of course it goes without saying that the terms of the series approach zero. If they did not, the series would not pass the n^{th} term test and the series would diverge.

Theorem 8.1 – The Alternating Series Test
If a_n is positive, the series $\sum(-1)^n a_n$ (or $\sum(-1)^{n+1} a_n$, etc.) converges if $\lim_{n\to\infty} a_n = 0$ and if $a_{n+1} < a_n$ for all n (at least past a threshold N).
Put another way, a series converges if the terms
1. strictly alternate,
2. decrease in magnitude, and
3. tend to zero.

Example 1
Use the alternating series test (AST) when applicable to determine which of the series in Practice 1 converge.

Practice 1
~~Which of the following series are alternating?~~

a. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!}$ b. $\sum_{n=0}^{\infty} (-3)^n$ c. $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n^2+1}$ d. $\cancel{\sum_{n=1}^{\infty} \sin(n)}$

$\frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}$ decreasing to zero! Yes

|Terms| go to ∞!

Decrease to zero ✓ Yes

Practice 2
Use the alternating series test, if applicable, to determine whether the following series converge.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ b. $\sum_{n=2}^{\infty} (-1)^n \cdot \frac{n}{n-1}$ c. $\sum_{n=1}^{\infty} (-1)^{n(n+1)/2} \cdot \frac{1}{n}$

a. Test applies $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$ Conv

b. Test applies $\lim_{n\to\infty} \frac{n}{n-1} = 1$ Diverges

c. Test doesn't apply Diverges

ALTERNATING SERIES ERROR BOUND

This table shows the errors when using partial sums to calculate the sum of the alternating harmonic series.

n	1	2	3	4	5	6	7	8	9	10
a_n	1	-0.5	0.3333	-0.25	0.2	0.8333	0.1429	-0.125	0.1111	-0.1
s_n	1	0.5	0.8333	0.58333	0.7833	0.6167	0.7595	0.6345	0.7456	0.6456
Actual Error	0.3069	0.1931	0.1402	0.1098	0.0902	0.0765	0.0664	0.0586	0.0525	0.0475
$\lvert a_{n+1}\rvert$	0.5	0.3333	0.25	0.2	0.8333	0.1429	0.125	0.1111	0.1	0.0909

Theorem 8.2 – Alternating Series Error Bound
If $\sum a_n$ is an alternating series in which $|a_{k+1}| < |a_k|$ for all k, then the error in the n^{th} partial sum is no larger than $|a_{n+1}|$. first omitted term

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Suppose I add up 49 terms of alt. harmonic
What is the error bound
No more than $\frac{1}{50} = .02$
I want an error of less than $= .0001$ $\frac{1}{10\,000}$
how many terms do I need?
If we do 9,999 terms, the error is .0001

Practice 2

Use the alternating series test, if applicable, to determine whether the following series converge.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ b. $\sum_{n=2}^{\infty} (-1)^n \cdot \frac{n}{n-1}$ c. $\sum_{n=1}^{\infty} (-1)^{n(n+1)/2} \cdot \frac{1}{n}$

Conv by AST

$\frac{n}{n-1} \to 1$

No

Doesn't alternate, test doesn't apply

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Theorem 8.2 – Alternating Series Error Bound

If $\sum a_n$ is an alternating series in which $|a_{k+1}| < |a_k|$ for all k, then the error in the n^{th} partial sum is no larger than $|a_{n+1}|$.

$$|S - s_n| < |a_{n+1}|$$

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

I want an error of less than .001

If I add up 999 terms

$$|S - s_{999}| < \frac{1}{1{,}000} = a_{1{,}000}$$

Theorem 8.2 – Alternating Series Error Bound

If $\sum a_n$ is an alternating series in which $|a_{k+1}| < |a_k|$ for all k, then the error in the n^{th} partial sum is no larger than $|a_{n+1}|$.

Example 3

Use s_5 and Theorem 8.2 to give bounds on the value of $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$.

$$1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} = .3\overline{6}$$

$$|S - .3\overline{6}| < \frac{1}{720}$$

Practice 3

Earlier, we claimed that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is roughly 0.693. How many terms are needed in the series to approximate this sum to these 3 decimal places?