HW: Page 77 # 27 - 39 (odd), 44 Page 93 #1 - 10

DO NOW:

Go over HW. Question numbers on the board.

DO NOW to discover the INTEGRAL TEST

What do we know already?

Taylor Series - how to write them for any function

Memorize them: Page 41, NOTE: these are polynomials

Power series converge on an interval

We find the radius of convergence using the ratio test

We need to learn <u>more</u> to figure out what happens at the endpts.

$\int_1^\infty \frac{1}{x^p}\,dx$ conv. if $p>1$

$\int_0^\infty e^{-x}\,dx$ conv.

Inf geom. series conv. if $|r|<1$

Practice 2

Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{nx^n}{4^n}$.

Radius=4

$$\sum_{n=1}^{\infty} \frac{nx^n}{4^n}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)x^{n+1}}{4^{n+1}} \cdot \frac{4^n}{nx^n}$$

$$= \frac{x}{4} \cdot \frac{n+1}{n}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x}{4}\right| \cdot \frac{n+1}{n} = \left|\frac{x}{4}\right| \qquad \left|\frac{x}{4}\right| < 1$$

If $\left|\frac{x}{4}\right| < 1$ this conv. $\longrightarrow$ $|x| < 4$

In Problems 26-40, find the radius of convergence of the given power series.

26) $1 + \frac{x}{5} + \frac{x^2}{25} + \frac{x^3}{125} + \cdots$ $\qquad a_n = \frac{x^n}{5^n}$

radius=5

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x}{5}\right|$$

What do we know about $\int_1^{\infty} \frac{1}{x}dx$ diverges

What can we conclude about $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges because it's bigger

What do we know about $\int_1^{\infty} \frac{1}{x^2}dx$ Converges

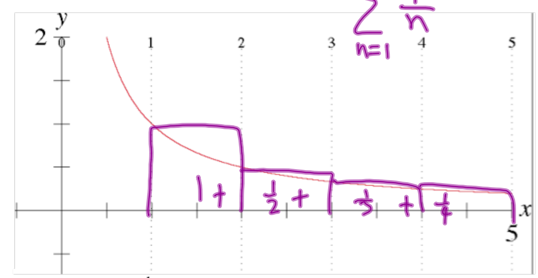
What can we conclude about $\sum_{n=2}^{\infty} \frac{1}{n^2}$ Converges

Math 504 DO NOW to discover the INTEGRAL TEST

1. Below is the graph of $y = \frac{1}{x}$.

Write a left-hand sum for $\int_1^{\infty} \frac{1}{x}dx$ with $\Delta x = 1$, then draw the rectangles in the graph.

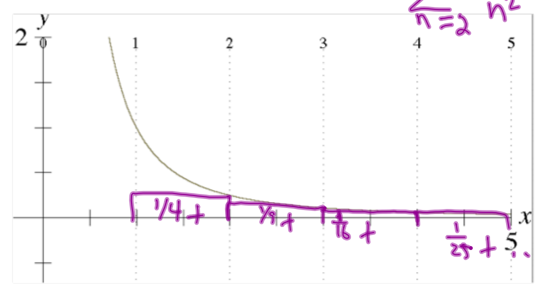
harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$



2. Below is the graph of $y = \frac{1}{x^2}$.

Write a right-hand sum for $\int_1^{\infty} \frac{1}{x^2}dx$ with $\Delta x = 1$, then draw the rectangles in the graph.

$\sum_{n=2}^{\infty} \frac{1}{n^2}$



The examples of H and S suggest that we can infer the behavior of a particular series by looking at a corresponding improper integral. This is stated precisely by the integral test.

Page 83

Theorem 7.1 – The Integral Test
If $f(x)$ is a positive, continuous, and decreasing function such that $f(n) = a_n$ for all n at least as large as some threshold value N, then $\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)dx$ either both converge or both diverge. In other words, as the improper integral behaves, so does the series, and vice versa.

The stuff about the threshold value N sounds complicated, but it is actually there to make things simpler. First, it gives us the freedom to not specify a starting index value in the theorem, which let's us be a bit more general. Second, if the function f doesn't satisfy all the hypotheses of the theorem (being positive, continuous, and decreasing) *until* some x-value, that's fine. As long as $f(x)$ matches the values of the series terms and eventually acts as needed, the conclusion of the theorem follows. Remember that convergence is not affected by the first "few" terms of a series, only by its long-run behavior, so we do not mind if f behaves erratically before settling down at $x = N$.

Example 1

Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges.

$\int_1^{\infty} \frac{x}{x^2+1}dx$

Series diverges

Use limit comparison with $\int_1^{\infty} \frac{1}{x}dx$, so diverges

Example 2

Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges. Series Converges

$\int_0^{\infty} \frac{1}{x^2+1}dx$ Converges

Practice 1

Determine whether the series $\sum_{n=2}^{\infty} \frac{n^2}{n^3-4}$ converges. divergence

$\int_1^{\infty} \frac{x^2}{x^3-4}dx$ like $\int_1^{\infty} \frac{1}{x}dx$

Radius of Convergence, Integral Test, p-series Test March 2, 2017

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Page 83

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Example 1

Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges.

$\int_1^{\infty} \frac{x}{x^2+1}dx$

limit comparison with $\frac{1}{x}$, integral diverges, so the series does

Example 2

Determine whether the series $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges. because $\int_0^{\infty} \frac{1}{x^2+1}dx$ conv.

Practice 1

Determine whether the series $\sum_{n=2}^{\infty} \frac{n^2}{n^3-4}$ converges

$\int_2^{\infty} \frac{x^2}{x^3-4}dx$

diverges

p - Series

After all this, I must admit that I try to avoid the integral test at all costs. The hypotheses can be a chore to verify, and the integration can also require quite a bit of effort. While some series might scream for the integral test, perhaps something like $\sum_{n=1}^{\infty} ne^{-n^2}$ is an example, in general the integral test is a test of last resort. Most series can be handled using simpler tests.

Why, then, did we spend so much time on the integral test? The integral test gives us a very simple and extremely useful convergence test as a consequence. First, though, we need a smidge of vocabulary.

Definition: A ***p*-series** is a series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ where p is a number. Page 86

The convergence of any p-series can be determined by using the integral test. $f(x) = \frac{1}{x^p}$ is positive and continuous for $x > 0$. And if p is positive, f is decreasing for $x > 0$. Thus we will be interested in computing $\int_1^{\infty} \frac{1}{x^p} dx$. But as we know from improper integrals, this integral converges

Theorem 7.2 – The *p*-series Test
A p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

Example 3
Determine whether the following series converge.

a. $\sum_{n=8}^{\infty} \frac{1}{\sqrt{n^3}}$ C

$p = 3/2$

b. $\sum_{n=1}^{\infty} \frac{1}{n^{0.1}}$ diverges

$p = 0.1$

Practice 2
Determine whether the following series converge.

a. $\sum_{n=1}^{\infty} \frac{1}{n^8}$ [redacted] C

b. $\sum_{n=1}^{\infty} \frac{\sqrt[4]{n}}{n}$ [redacted] D

c. $\sum_{n=1}^{\infty} n^{-1/3}$ D [redacted]

Extend Page