HW: Page 93 #16 - 19, 21 - 33 (odd)

DO NOW:

On your own (no help, no notes) find the following antiderivatives:

$$\int \frac{1}{x^2}dx = \int x^{-2}dx = -x^{-1} + C$$

$$\int e^{-2x}dx = -\frac{1}{2}e^{-2x} + C$$

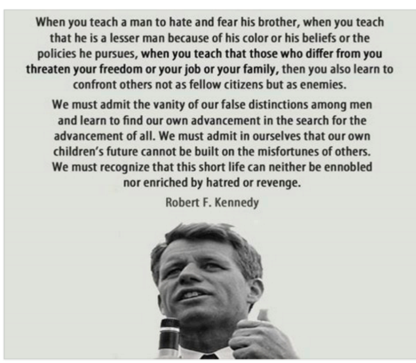
$$\int x\sin(x^2)dx = \frac{1}{2}\int \sin(u)\,du = +\frac{1}{2}(-\cos(u)) + C = -\frac{1}{2}\cos(x^2) + C$$

$u = x^2$ $\quad du = 2x\,dx$ $\quad \frac{1}{2}du = x\,dx$

$$\int \frac{1}{x\ln x}dx = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$$

$u = \ln x$

$du = \frac{1}{x}dx$



9 $\quad \sum \frac{n^2}{\sqrt{n^3 \cancel{+2}}}$

36 $\quad \sum \frac{(\cos n\pi)(x+2)^n}{3^n}$

$(\cos n\pi)$ $\quad n = 0, 1, 2, 3, 4$

$\quad 1, -1, 1, -1, 1$

37 $\quad \sum (-1)^{n+1}\frac{(x-4)^{2n}}{4n}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-4)^{2n+2}}{\cancel{4}(n+1)} \cdot \frac{\cancel{4}n}{(x-4)^{2n}}\right|$$

$$= \left|\frac{n}{n+1}(x-4)^2\right|$$

$$\lim_{n\to\infty} \frac{n}{n+1}(x-4)^2 = (x-4)^2$$

$(x-4)^2 < 1$ $\quad$ Radius $= 1$

$|x-4| < 1$

p. 73

58 $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = e^x$

$$\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \right| =$$

$\displaystyle\lim_{n\to\infty} \frac{1}{n+1}|x| = 0$ Radius is infinite

p. 87 $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \cdots \frac{1}{4^n} + \cdots$

$\int 4^{-x}\,dx$

Theorem 7.3 – The Direct Comparison Test

Suppose $a_n \geq 0$ and $b_n \geq 0$ for all n (or at least all n past some threshold N). Further suppose that $a_n \leq b_n$ for all n (or, again, all n past N).

If $\sum_{n=N}^{\infty} b_n$ converges, then $\sum_{n=N}^{\infty} a_n$ converges.

If $\sum_{n=N}^{\infty} a_n$ diverges, then $\sum_{n=N}^{\infty} b_n$ diverges.

If either $\sum_{n=N}^{\infty} a_n$ converges or $\sum_{n=N}^{\infty} b_n$ diverges, then no conclusion can be drawn about the other series based on this test.

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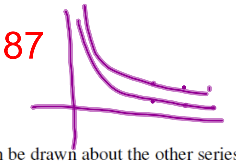

Example 4

Determine whether the following series converge. $\sum \frac{1}{n}$

a. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+5}$ b. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$ div c. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$

conv, $\sum \frac{1}{n^2}$

$\frac{1}{a} < \frac{1}{b}$ $(a, b > 0)$

n^2+5 if n^2

$a > b$

n^2+5 n^2

$\frac{1}{n} < \frac{1}{\ln n}$

$n > \ln n$ for all $n \geq 1$

looks like

$\sum \frac{1}{3^n} = \sum \left(\frac{1}{3}\right)^n$

geom $r = \frac{1}{3}$ conv.

Theorem 7.4 – The Limit Comparison Test

Suppose $a_n > 0$ and $b_n > 0$ for all n (or at least all n past a certain threshold N).

If $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists and is both positive and finite, then $\sum_{n=N}^{\infty} a_n$ and $\sum_{n=N}^{\infty} b_n$ either both converge or both diverge.

not zero

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$\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$ Both conv.

$\lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{3^n - 1}} = 1$

$-.000001$ within $+.0000005$

Practice 4

Determine whether the series $\sum_{n=2}^{\infty} \frac{n^2 - 1}{n^3}$ converges. diverges

Compare with $\sum \frac{1}{n}$

$\lim_{n\to\infty} \frac{\frac{n^2-1}{n^3}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^3 - n}{n^3} = 1$

What Convergence Tests do we have for Positive Series?

There are 7:

1. Nth term test, div if $\lim_{n\to\infty} a_n \neq 0$ Can't determine conv.
2. Geometric, conv if $|r| < 1$, otherwise div.
3. Ratio Test $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$ conv if $L < 1$, div if $L > 1$, ?? if $L = 1$
   Use with factorials and exponential functions
5. p-Series $\sum \frac{1}{n^p}$ conv. if $p > 1$, otherwise div.
4. Integral test $\int f(x)dx$ and $\sum f(n)$ both converge or both diverge
6. Direct Comparison
7. Limit Comparison

To be successful with series, you need to develop a sense for when to use each test.

Geometric and p series announce themselves fairly clearly, so there should never be any doubt as to when to apply those tests.
The ratio test is great when dealing with factorials and exponential factors (and also power series, though never the endpoints).
However, the ratio test is terrible for p-series and p-like series.
Convergence of a series can be ruled out with the nth term test, but we can never show convergence with it.
If the general term looks like something you can integrate, there is always the integral test, though the comparison tests should often be considered before going there.

Two Practice AP Tests

They count for fourth quarter.
They will be curved.
We will do some in-class review next week and the following week.
The goal is PRACTICE and experiencing a long challenging test on the whole course.