Taylor Poly fit for sin 3_1_12-1.ggb

DO NOW:

Questions from HW on board.

Find linear functions f and g such that $f(0)=5$, $f'(0)=-3$, $g(2)=-4$, and $g'(2)=\frac{1}{3}$

Read pages 19 and 20.
We will be looking at a geogebra file to examine this problem.

$$\int_1^\infty \frac{dx}{x^2} \quad C$$

$$\int_0^\infty \frac{dx}{e^x} \quad C$$

$$\int_{-1}^1 \frac{dx}{\sqrt[3]{x}}$$

$$\int_{-1}^1 \frac{dx}{x^2}$$

$$\boxed{\int_0^1 \frac{1}{x^p}\,dx = \begin{cases} \text{Conv if } p<1 \\ \text{Div if } p\geq 1 \end{cases}}$$

$$\int_0^1 \frac{1}{x^{1/3}}\,dx =$$

$$\lim_{t\to 0^+} \int_t^1 \frac{1}{x^{\frac{1}{3}}}\,dx =$$

$$\lim_{t\to 0^+} \frac{3}{2}x^{\frac{2}{3}}\Big]_t^1 =$$

$$\frac{3}{2}$$

DO NOW:

Questions from HW on board.

Find linear functions f and g such that $f(0)=5$, $f'(0)=-3$, $g(2)=-4$, and $g'(2)=\frac{1}{3}$

Re-read pages 19 and 20.

We will be looking at a geogebra file to examine this problem.

$f(x)=5+(-3)x$

$g(x)=-4+\frac{1}{3}(x-2)$

$$\sin x \approx x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}. \qquad (1)$$

p. 22

We can look in the chapter to see graphs that show how well these polynomials match sin(x).

p. 23

The polynomials that we have been developing and graphing are called Taylor Polynomials after English mathematician Brook Taylor (1685-1731). Every Taylor polynomial has a center which in the case of our example has been $x = 0$. When a Taylor polynomial is centered at (or expanded about) $x = 0$, we sometimes call it a Maclaurin Polynomial after Scottish mathematician Colin Maclaurin (1698-1746).

Observation: Taylor Polynomials...

1. … match the function being modeled perfectly at the center of the polynomial.
2. … lose accuracy as we move away from the center.
3. … gain accuracy as we add more terms.

New Polynomials from Old

Suppose we also want a Maclaurin polynomial for the cosine function. We could start over from scratch. That would involve starting with the linearization function $y = 1$ and adding terms, one at a time, hoping to hit upon something that looks good. But this seems like a lot of work. After all, the cosine function is the *derivative* of the sine function. Maybe we can just differentiate the Maclaurin polynomials.

$$\sin x \approx x-\frac{x^3}{6}+\frac{x^5}{120}$$

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

$$\cos x \approx 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!} - \cdots + (-1)^n \frac{(2n+1)x^{2n}}{(2n+1)!}$$

$$\approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\frac{\sin x}{x} \approx \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!}}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} 1 - \frac{x^2}{3!} + \frac{x^4}{5!} = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} \overset{L}{=} \lim_{x\to 0} \frac{\cos x}{1} = 1$$

Intro to Taylor Polynomials February 16, 2017

Page 34

Find f if $f(0)=5$ and $f'(0)=-3$ and f is linear

$f(x) = 5 + (-3)x$

Find g if $g(2)=7$ and $g'(2)=\frac{2}{3}$ and g is linear

$g(x) = 7 + \frac{2}{3}(x-2)$

Find quadratic $h(x)$ where $h(2)=8$ $h'(2)=-1$ and $h''(2)=3$

$h(x) = 8 - 1(x-2) + \frac{3}{2}(x-2)^2$ $h(2)=8$

$h'(x) = -1 + \frac{6}{2}(x-2)$ $h'(2)=-1$

$h''(x) = \frac{6}{2}$ $h''(2) = \frac{6}{2}$

Find a cubic $j(x)$ where $j(0)=-1$, $j'(0)=5$, $j''(0)=\frac{1}{5}$, $j'''(0)=-12$