Intro to Taylor Polynomials

February 16, 2017

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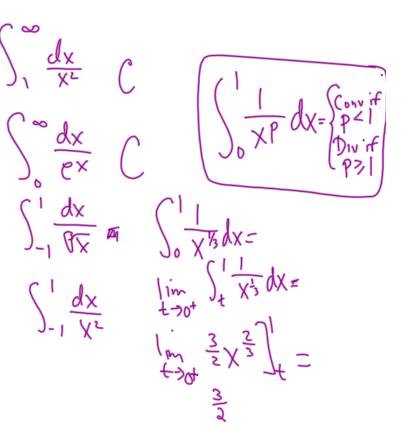
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Taylor Poly fit for sin 3\_1\_12-1.ggb

## DO NOW: Questions from HW on board.

Find linear functions f and g such that f(0) = 5, f'(0) = -3, g(2) = -4, and  $g'(2) = \frac{1}{3}$ 

Read pages 19 and 20. We will be looking at a geogebra file to examine this problem.



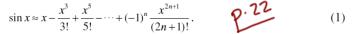
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F(x)=5+(-3)x $g(x)=-4+\frac{1}{3}(x-2)$ 



We can look in the chapter to see graphs that show how well

## these polynomials match sin(x).

The polynomials that we have been developing and graphing are called Taylor Polynomials after English mathematician Brook Taylor (1685-1731). Every Taylor polynomial has a center which in the case of our example has been x = 0. When a Taylor polynomial is centered at (or expanded about) x = 0, we sometimes call it a Maclaurin Polynomial after Scottish mathematician Colin Maclaurin (1698-1746).

Observation: Taylor Polynomials...

- 1. ... match the function being modeled perfectly at the center of the polynomial.
- 2. ... lose accuracy as we move away from the center.
- 3. ... gain accuracy as we add more terms.

## New Polynomials from Old

Suppose we also want a Maclaurin polynomial for the cosine function. We could start over from scratch. That would involve starting with the linearization function y = 1 and adding terms, one at a time, hoping to hit upon something that looks good. But this seems like a lot of work. After all, the cosine function is the *derivative* of the sine function. Maybe we can just differentiate the Maclaurin polynomials.

$$\sin x \approx x - \frac{x^3}{6} + \frac{x^5}{120}$$

$$\sin x \approx x - \frac{x^{3}}{3!} + \frac{x^{5}}{5!} - \dots + (-1)^{n} \frac{x^{2n+1}}{(2n+1)!}.$$

$$\cos x \approx \left| -\frac{3\chi^{2}}{3!} + \frac{5\chi^{4}}{5!} - \dots + (-1)^{n} \frac{(2n+1)\chi^{2n}}{(2n+1)!} \right|$$

$$\approx \left| -\frac{\chi^{2}}{2!} + \frac{\chi^{4}}{4!!} - \dots + (-1)^{n} \frac{\chi^{2n}}{(2n)!} \right|$$

$$\sum_{x \to 0}^{\infty} \frac{\chi - \chi^{3}}{3!} + \frac{\chi^{5}}{5!} = \left| -\frac{\chi^{2}}{3!} + \frac{\chi^{4}}{5!} \right|$$

$$\sum_{x \to 0}^{\infty} \frac{51N\chi}{\chi} = \lim_{x \to 0}^{\infty} \left| -\frac{\chi^{2}}{3!} + \frac{\chi^{4}}{5!} \right| = 1$$

$$\frac{Page 34}{Find f if f(0)=5 and f'(0)=3}$$
  
and f is linear  
 $f(x)=5+(-3)x$   
Find gif  $g(2)=7$  and  $g'(2)=\frac{2}{3}$  and gislness  
 $g(x)=7+\frac{2}{3}(x-2)$   
Find quadratic  $h(x)$  where  
 $h(2)=8$   $h'(2)=-1$  and  $h''(2)=3$   
 $h(x)=8-1(x+2)+\frac{3}{2}(x-2)^2$   $h(2)=\frac{6}{3}$   
 $h'(x)=-1+\frac{6}{2}(x-2)$   $h'(2)=-1$   
 $h''(x)=\frac{6}{2}$   $h''(2)=\frac{6}{3}$   
Find a cubic  $j(x)$  where  $j(0)=-1, j'(0)=5$   
 $j''(0)=-\frac{1}{5}$   $j''(0)=-12$ 

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