HW: Note Card is due Friday
Finish Practice MC for this unit, please show work!
Suggested Review: Limit Comparison test practice on improper integrals.

DO NOW:

Questions from the HW?
With your group do page 16 #42-46 (instructions are at the bottom of page 15)

Upcoming Tests:

Friday, Feb. 17: Improper Integrals, L'Hopital's Rule, Sequences and Series Intro

Tuesday, March 21: Practice AP Free Response

Wednesday, March 22: Practice AP Non-calculator Multiple Choice

These two tests in March are during the ELA MCAS exam week

THINGS TO REMEMBER

If $\lim_{x\to\infty} f(x) = 0$, $\int_a^\infty f(x)\,dx$ MIGHT converge

If $\lim_{n\to\infty} a_n = 0$, $\sum_{n=0}^\infty a_n$ MIGHT converge.

$$\int_1^\infty \frac{1}{x^p}\,dx \begin{cases} p>1 & \text{converges} \\ p=1 & \text{diverges} \\ p<1 & \text{diverges} \end{cases}$$

$\int_0^\infty e^{-x}\,dx$ converges

Page 16 #42-46

For problems 42-47 indicate whether the statement is True or False. Support your answer with reasons and/or counterexamples

42. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

False Think harmonic series

43. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

True, n^{th} term test

44. If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} s_n = 0$.

False, partial sums don't have to go to zero

45. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

False $\lim_{n\to\infty} a_n = 0$, so $\lim_{n\to\infty} \frac{1}{a_n} \neq 0$

46. If $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges, then $\sum_{n=1}^{\infty} a_n$ converges.

False let $a_n = n$

So $\sum \frac{1}{a_n}$ diverges and $\sum_{n=1}^{\infty} n$ diverges.

math.arizona.edu/~calc/Text/Section7.8.pdf

Use the limit comparison test to decide whether the following improper integrals converge

1. $\int_1^{\infty} \frac{x^2}{x^4+1}\,dx$ C $\lim_{x\to\infty} \frac{\frac{1}{x^2}}{\frac{x^2}{x^4+1}} = 1$
2. $\int_2^{\infty} \frac{x^3}{x^4-1}\,dx$ $\frac{1}{x}$ D
3. $\int_1^{\infty} \frac{x^2+1}{x^3+3x+2}\,dx$ $\frac{1}{x}$ D
4. $\int_1^{\infty} \frac{1}{x^2+5x+1}\,dx$ $\frac{1}{x^2}$ C
5. $\int_1^{\infty} \frac{x}{x^2+2x+4}\,dx$ $\frac{1}{x}$ D
6. $\int_1^{\infty} \frac{x^2-6x+1}{x^2+4}\,dx$ $\frac{1}{x^0}$ D
7. $\int_1^{\infty} \frac{5x+2}{x^4+8x^2+4}\,dx$ $\frac{1}{x^3}$ C
8. $\int_1^{\infty} \frac{1}{e^{5t}+2}\,dt$ e^{-t} C
9. $\int_1^{\infty} \frac{x^2+4}{x^4+3x^2+11}\,dx$ $\frac{1}{x^2}$ C
10. $\int_{50}^{\infty} \frac{dz}{z^3}$
11. $\int_1^{\infty} \frac{dx}{1+x}$
12. $\int_1^{\infty} \frac{dx}{x^3+1}$
13. $\cancel{\int_5^{8} \frac{6}{\sqrt{t-5}}\,dt}$
14. $\cancel{\int_0^{1} \frac{1}{x^{19/20}}\,dx}$
15. $\cancel{\int_{-1}^{5} \frac{dt}{(t+1)^2}}$
16. $\cancel{\int_{-\infty}^{\infty} \frac{du}{1+u^2}}$
17. $\int_1^{\infty} \frac{du}{u+u^2}$
18. $\int_1^{\infty} \frac{d\theta}{\sqrt{\theta^2+1}}$ $\frac{1}{\theta}$ D
19. $\int_2^{\infty} \frac{d\theta}{\sqrt{\theta^3+1}}$ $\frac{1}{\theta^{1.5}}$ C

$\int^{\infty} \frac{1}{x^p}\,dx =$

Conv if $p>1$
div if $p \le 1$

Greatest Hits

Improper Integrals, L'Hopital's Rule, Sequences and Series Intr

① Improper Integrals

a) Evaluate

b) Determine convergence using comparison/limit comparison (Don't need to show work)

② L'Hopital's Rule - finding $\frac{0}{0}$ or $\frac{\infty}{\infty}$ limits

③ Sequences - Converge?, finding limits

④ Series

a) Geometric Series - find the sum unless $|r| \geq 1$

b) Telescoping Series - find the sum

c) N^{th} term test - determines divergence

These definitions are needed for review sheet NOT test

Problem 8) definitions:

monotonic: strictly increasing, strictly decreasing

not monotonic

bounded: terms don't get bigger or smaller than a given #

$m < a_n < M$

Unbounded = not bounded

Format of Test: NO CALCULATOR

1) Converge/diverge questions for improper integrals, series

(1 point problems)

2) Short answer questions on improper integrals, L'Hopital's rule, limits of sequences and series:

Work must be shown and will be graded for correctness, partial credit available

(3 point problems)

3) NO open response questions

What does the Fundamental Theorem of Calculus say about the derivative of

$\int_1^x f(t)dt$ with respect to x

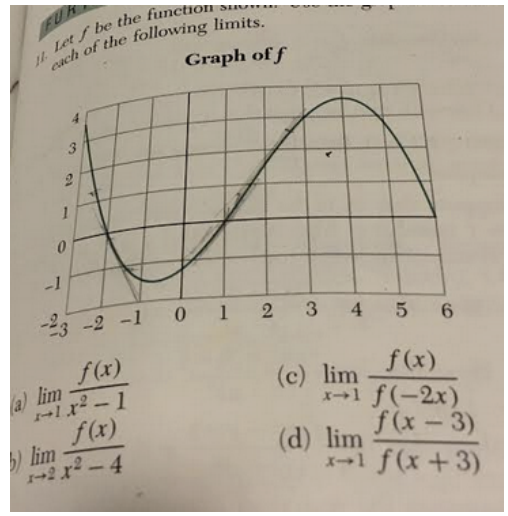
$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

(37) $$\lim_{x\to\infty} \frac{\int_1^x \sqrt{1+e^{-3t}}\,dt}{x} \overset{L}{=}$$

$$\lim_{x\to\infty} \frac{\sqrt{1+e^{-3x}}}{1} = 1$$

(39) $$\lim_{x\to\infty} e^{x^2}\int_0^x e^{-t^2}dt = \infty$$

(40) Use L'Hopital


11. Let f be the function shown ... each of the following limits.

(a) $\lim_{x\to1}\frac{f(x)}{x^2-1}$

(b) $\lim_{x\to2}\frac{f(x)}{x^2-4}$

(c) $\lim_{x\to1}\frac{f(x)}{f(-2x)}$

(d) $\lim_{x\to1}\frac{f(x-3)}{f(x+3)}$

a) $$\lim_{x\to1}\frac{f(x)}{x^2-1} \overset{L}{=}$$

$$\lim_{x\to1}\frac{f'(x)}{2x} \approx \frac{2}{2} = 1$$

b) $$\lim_{x\to2}\frac{f(x)}{x^2-4}$$

DNE

Greatest Hits

1. Improper Integrals
   a) Evaluate
   b) Determine convergence using comparison/limit comparison (Don't need to show work)
2. L'Hopital's Rule - finding $\frac{0}{0}$ or $\frac{\infty}{\infty}$ limits
3. Sequences - Converge?, finding limits
4. Series
   a) Geometric Series - find the sum unless $|r| \geq 1$
   b) Telescoping Series - find the sum
   c) N^{th} term test - determines divergence
5. Taylor Polynomials = know the formula
   a) Given derivs, write the polynomial
   b) Given the polynomial, determine derivs
   c) Find a Taylor polynomial by diff an easy function, be careful if it's not centered at zero.

These definitions are needed for review sheet NOT test

8) definitions:
   monotonic: strictly increasing / strictly decreasing
   bounded: terms don't get bigger or smaller than a given #
   $m < a_n < M$
   unbounded = not bounded