HW:
Series Packet: Page 14 #7- 10, 12- 14, 22, 24, 35-39 (all)

DO NOW:

Use Solution Manual to check answers and solutions.
Which ones do we need to go over?

Problems from L'Hopital: Volunteers to put up 11, 37, 39?

$$\int_0^\infty e^{-10x}\,dx = \lim_{t\to\infty}\int_0^t e^{-10x}\,dx$$

$$= \lim_{t\to\infty} -\frac{1}{10}e^{-10x}\Big]_0^t$$

$$\frac{d\left(-10e^{-10x}\right)}{dx} = 100e^{-10x}$$

$$= \lim_{t\to\infty}\left(-\frac{1}{10}e^{-10t} - \left(-\frac{1}{10}e^{-10(0)}\right)\right)$$

$$= \frac{1}{10}$$

$$\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} \text{converges to } \frac{1}{p-1} & \text{if } p>1 \\ \text{diverges} & \text{if } p\le 1 \end{cases}$$

If f and g are positive, continuous on $x \ge a$
and if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ with $0<L<\infty$
then $\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ either both conv or both diverge.

$\frac{\sqrt[3]{x}}{\sqrt[4]{(x+1)^5}}$ Looks like $\frac{x^{\frac{1}{3}}}{x^{\frac{5}{4}}}$

Definition A series **converges** if the sequence of partial sums converges. That is, we say that $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{n\to\infty} s_n$ converges, where $s_n = a_1 + a_2 + a_3 + \cdots + a_n$. If the sequence of partial sums does not converge, we say that the series **diverges**. If the series converges, the limit of s_n is called the sum or value of the series.

Practice 5

$2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

For the series $\sum_{n=1}^{\infty} \frac{1}{3^n}$, list the first five terms of the series and the first five partial sums of the series. Does the series appear to converge?

$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \cdots$

$S_1 = \frac{1}{3}$ $S_2 = \frac{4}{9}$ $S_3 = .48148$ $S_4 = .49383$ $S_5 = .49794$

Theorem 1.1

If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then…

1. $\sum_{n=1}^{\infty} c \cdot a_n = c \cdot \sum_{n=1}^{\infty} a_n = cA$
2. $\sum_{n=1}^{\infty} (a_n \pm b_n) = A \pm B$
3. for any positive integer k, $\sum_{n=k}^{\infty} a_n$ converges, though almost certainly not to A.

Practice 6

Compute several partial sums for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Does the series seem to converge? To what?

Example 6

Compute several partial sums for the series $\sum_{n=1}^{\infty} \sqrt{n}$. Does the series seem to converge?

Theorem 1.2 – The n^{th} Term Test

In order for a series to converge, it is necessary that the parent sequence converges to zero. That is, given a series $\sum a_n$, if $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.

It may be surprising to you that the converse is *not* a true statement.

The most famous example of this is the so-called harmonic series:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots$$

To summarize: If the terms of a series do not go to zero as n→∞, then the series diverges. But if the terms do go to zero, that does not necessarily mean that the series will converge. The nth term test cannot show convergence of a series.

Write out the terms

23) $\sum_{n=1}^{\infty} \frac{5}{3^n} = \frac{5}{3}+\frac{5}{9}+\frac{5}{27}+\dots$

Formula $= \frac{\text{First term}}{1-\text{ratio}} = \frac{a}{1-r}$

$= \frac{\frac{5}{3}}{1-\frac{1}{3}}$

Telescoping Series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right) = \ldots\ldots$$

$$S_4 = \left(1-\cancel{\tfrac{1}{2}}\right)+\left(\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{3}}\right)+\left(\cancel{\tfrac{1}{3}}-\cancel{\tfrac{1}{4}}\right)+\left(\cancel{\tfrac{1}{4}}-\tfrac{1}{5}\right)$$

$$= 1-\frac{1}{5} = \frac{4}{5}$$

$$S_n = 1-\frac{1}{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 1-\frac{1}{n+1} = 1$$

To determine if a series converges look at the sequence of partial sums. If $\lim_{n\to\infty} S_n$ is finite, that is the sum of the series.