HW: Page 549 #3 - 39 (odd)

DO NOW:

Use a table on the calculator to "guess" each of the following limits:

$$\lim_{x\to\infty}\frac{x^2}{2^x} = 0$$

$$\lim_{x\to\infty}\frac{x^2}{(3x+10)^2} = \frac{1}{9}$$

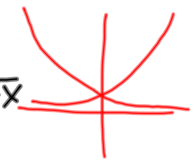
$$\lim_{x\to\infty}\frac{2^x}{(3x+10)^2} = \infty$$

Dalai Lama

1 hr ·

In my experience, what we need is a calm mind and warm-heartedness provides a basis for that. That's how we make ourselves happy as individuals in families, local communities and nations. I believe that if we can train those who are young today in these qualities the world will be a more peaceful place later in this century.

(21) $\int_0^\infty \frac{dx}{x+e^{-x}}$



$\int_1^\infty \frac{1}{x^p}\,dx$

$\frac{1}{2x} < \frac{1}{x+e^{-x}}$ $\quad x+e^{-x} < 2x$ $\quad \int_0^\infty e^{-x}\,dx$

Diverges, bigger than $\frac{1}{2x}$

(23) $\int_0^\infty \frac{\sqrt{x}}{x+1}\,dx$ looks like $\frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}}$

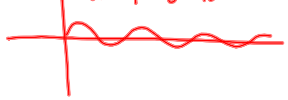
diverges

(36) $\int_{-\infty}^{\infty} e^{-x}\,dx = \int_{-\infty}^{0} e^{-x}\,dx + \int_0^\infty e^{-x}\,dx$

Div. Conv.

One more detail from Improper Integrals:

$\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$



What if f(x) is not always positive?
We look at |f(x)| and see what happens.
If the improper integral converges for |f(x)|, it will converge for f(x), which will be less.

Theorem on Absolute Convergence:

Let f and g be continuous functions such that for all $x \geq a$,
$0 \leq |f(x)| \leq g(x)$

If $\int_a^\infty g(x)dx$ converges, then $\int_a^\infty f(x)dx$ also converges

and $\left|\int_a^\infty f(x)dx\right| < \int_a^\infty g(x)dx$

Example $\int_0^\infty \frac{\sin x}{x^2}dx$ compare to

$\int_0^\infty \frac{1}{x^2}dx$

since $|\sin x| \leq 1$

L'Hopital's Rule
(for some limits involving ∞)

This rule will help us figure out limits when both terms in a quotient are getting big or small, which term "gets there faster"

This rule only works for indeterminate limits that look like either

$$\frac{\infty}{\infty} \quad \text{or} \quad \frac{0}{0}$$

EXAMPLES:

$$\lim_{x\to 0}\frac{\sin(2x)}{x} \qquad \left(\frac{0}{0}\right)$$

$$\lim_{x\to\infty}\frac{x^2}{2^x} \qquad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty}\frac{x}{e^x} \qquad \left(\frac{\infty}{\infty}\right)$$

This looks like a product, but we can rewrite it as a quotient

L'Hopital's Rule

Let f and g be differentiable and suppose as $x \to a$, either:

1) $f(x) \to 0$ AND $g(x) \to 0$

OR

2) $f(x) \to \pm\infty$ AND $g(x) \to \pm\infty$

AND $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists

THEN: $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

NOTE: Take the derivatives separately.
REMEMBER THE QUOTIENT RULE IS DIFFERENT!

EXAMPLES:

$$\lim_{x \to 0} \frac{\sin(2x)}{x} \overset{L}{=} \lim_{x \to 0} \frac{2\cos(2x)}{1} = \frac{2}{1} = 2$$

$$\lim_{x \to \infty} \frac{x^2}{2^x} \overset{L}{=} \lim_{x \to \infty} \frac{2x}{(\ln 2)2^x} \overset{L}{=} \lim_{x \to \infty} \frac{2}{\underbrace{(\ln 2)^2 2^x}_{\to\infty}} \to 2 = 0$$

$$\lim_{x \to \infty} xe^{-x} = \lim_{x \to \infty} \frac{x}{e^x} \overset{L}{=} \lim_{x \to \infty} \frac{1}{\underbrace{e^x}_{\to\infty}} = 0.$$

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\overset{\to -\infty}{\ln x}}{\underbrace{\frac{1}{x}}_{\to\infty}}$$

$$\overset{L}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$$

$$= \lim_{x \to 0^+} \left(\frac{1}{x}\right)\left(\frac{-x^2}{1}\right)$$

$$= \lim_{x \to 0^+} -x$$

$$= 0$$

HW: Page 549 #3 - 39 (odd)

So, the rate at which the functions go to infinity, from

slowest → fastest

logarithmic → polynomials → exponential

Which of these integrals are improper?

a) $\int_0^1 \frac{\sin x}{x}\,dx$ NOT $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Which integrands have a finite limit at zero?

b) $\int_0^1 \frac{\cos x}{x}\,dx$ $\lim_{x\to 0^+} \frac{\cos x}{x} = \infty$

c) $\int_0^1 x\ln x\,dx$ NOT $\lim_{x\to 0^+} x\ln x = 0$

d) $\int_0^1 \frac{\arctan x}{\sqrt[3]{x}}\,dx$

L'Hopital