HW: No written HW. I will post some videos on our next topic.

DO NOW:

Go over HW with your group.

Put together a list of what you are not sure you understand (or what you're sure you don't understand yet) from this unit.

We will spend this class trying to develop better understanding.

On a computer, or your phone, go to Desmos.com and open the calculator.

(36) $\int_{-\infty}^{\infty} e^{-x}\,dx$

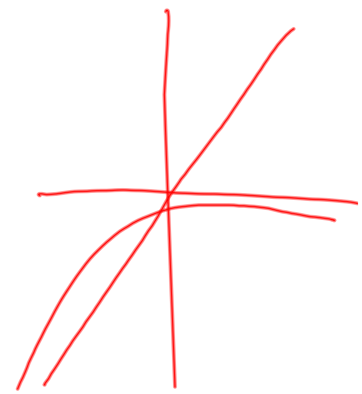
Each impropriety requires a separate integral and a separate limit.

$\underbrace{\int_{-\infty}^{0} e^{-x}\,dx}_{\text{div}} + \underbrace{\int_{0}^{\infty} e^{-x}\,dx}_{\text{conv.}}$

(21) $\int_{0}^{\infty} \frac{dx}{x+e^{-x}}$ $=0$ if

$x=-e^{-x}$?

when x=A



http://www.math.uri.edu/~pakula/142sec3f11/limcomp.pdf

$$\lim \frac{\frac{1}{x^6}}{\frac{1}{x^5+1}} = 0$$

Useless

The Limit Comparison Test for Improper Integrals

The following test is often, but not always, a useful alternative to the comparison test given on p. 381 of the textbook.

Limit Comparison Test. If $f(x)$ and $g(x)$ are both positive when $x > a$ and

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L \quad \text{and} \quad 0 < L < \infty$$

Finite # Positive

then the improper integrals

$$\int_a^\infty f(x)\,dx \quad \text{and} \quad \int_a^\infty g(x)\,dx$$

are either *both convergent* or *both divergent*.

$$\int \frac{1}{x^5+1}\,dx$$

Choose $\frac{1}{x^5}$

$$\lim_{x\to\infty} \frac{\frac{1}{x^5}}{\frac{1}{x^5+1}} = \qquad \lim_{x\to\infty} \frac{x^5+1}{x^5} = 1$$

Example. Is $\int_2^\infty \frac{1}{\sqrt{x+1}}\,dx$ convergent? The obvious integral to use for comparison is $\int_2^\infty \frac{1}{\sqrt{x}}\,dx$ which we know diverges because $\int_2^\infty \frac{1}{x^p}\,dx$ diverges when $p < 1$. However,

$$\frac{1}{\sqrt{x+1}} \le \frac{1}{\sqrt{x}}$$

so the obvious comparison is not the one we want. (We would want the opposite inequality!) On the other hand, using the Limit Comparison Test we find

$$\lim_{x\to\infty} \frac{\frac{1}{\sqrt{x+1}}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \sqrt{\frac{x}{x+1}} = \sqrt{1} = 1.$$

Here $L = 1$ and $0 < 1 < \infty$ so we conclude that our integral is divergent.

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What to compare to?

$f(x) = \frac{1}{x^{10}+3x^2}$ looks like $g(x) = \frac{1}{x^{10}}$

$h(x) = \frac{1}{e^{-x}+x^2}$ looks like $m(x) = \frac{1}{x^2}$

$q(x) = \frac{1}{(\sqrt{x-1})(x^3+5)}$ $\qquad r(x) = \frac{1}{x^{7/2}}$

$n(x) = \frac{\sqrt{x}}{x^2-10^{23}}$ $\qquad c(x) = \frac{1}{x^{3/2}}$

math.arizona.edu/~calc/Text/Section7.8.pdf

Use the limit comparison test to decide whether the following improper integrals converge

1. $\int_1^\infty \frac{x^2}{x^4+1}dx$
2. $\int_2^\infty \frac{x^3}{x^4-1}dx$
3. $\int_1^\infty \frac{x^2+1}{x^3+3x+2}dx$ $\quad \frac{x^2}{x^3}=\frac{1}{x}$ compare to $\int_1^\infty \frac{1}{x}dx$ div
4. $\int_1^\infty \frac{1}{x^2+5x+1}dx$
5. $\int_1^\infty \frac{x}{x^2+2x+4}dx$
6. $\int_1^\infty \frac{x^2-6x+1}{x^2+4}dx$
7. $\int_1^\infty \frac{5x+2}{x^4+8x^2+4}dx$
8. $\int_1^\infty \frac{1}{e^{5t}+2}dt$ $\quad$ Compare to $\int_1^\infty e^{-5t}dt$
9. $\int_1^\infty \frac{x^2+4}{x^4+3x^2+11}dx$
10. $\int_{50}^\infty \frac{dz}{z^3}$
11. $\int_1^\infty \frac{dx}{1+x}$
12. $\int_1^\infty \frac{dx}{x^3+1}$
13. $\cancel{\int_5^8 \frac{6}{\sqrt{t-5}}dt}$
14. $\cancel{\int_0^1 \frac{1}{x^{19/20}}dx}$
15. $\cancel{\int_{-1}^5 \frac{dt}{(t+1)^2}}$
16. $\int_{-\infty}^\infty \frac{du}{1+u^2}$
17. $\int_1^\infty \frac{du}{u+u^2}$
18. $\int_1^\infty \frac{d\theta}{\sqrt{\theta^2+1}}$
19. $\int_2^\infty \frac{d\theta}{\sqrt{\theta^3+1}}$

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Use the limit comparison test to decide whether the following improper integrals converge

$$\lim_{x\to\infty} \frac{\frac{1}{x^2}}{\frac{x^2}{x^4+1}} = 1$$

→ 1. $\int_1^\infty \frac{x^2}{x^4+1}dx$ C
→ 2. $\int_2^\infty \frac{x^3}{x^4-1}dx$ $\frac{1}{x}$ D
3. $\int_1^\infty \frac{x^2+1}{x^3+3x+2}dx$ $\frac{1}{x}$ D
4. $\int_1^\infty \frac{1}{x^2+5x+1}dx$ $\frac{1}{x^2}$ C
5. $\int_1^\infty \frac{x}{x^2+2x+4}dx$ $\frac{1}{x}$ D
6. $\int_1^\infty \frac{x^2-6x+1}{x^2+4}dx$ $\frac{1}{x^0}$ D
7. $\int_1^\infty \frac{5x+2}{x^4+8x^2+4}dx$ $\frac{1}{x^3}$ C
8. $\int_1^\infty \frac{1}{e^{5t}+2}dt$ e^{-t} C
9. $\int_1^\infty \frac{x^2+4}{x^4+3x^2+11}dx$ $\frac{1}{x^2}$ C
10. $\int_{50}^\infty \frac{dz}{z^3}$
11. $\int_1^\infty \frac{dx}{1+x}$
12. $\int_1^\infty \frac{dx}{x^3+1}$
13. $\cancel{\int_5^8 \frac{6}{\sqrt{t-5}}dt}$
14. $\cancel{\int_0^1 \frac{1}{x^{19/20}}dx}$
15. $\cancel{\int_{-1}^5 \frac{dt}{(t+1)^2}}$
16. $\cancel{\int_{-\infty}^\infty \frac{du}{1+u^2}}$
17. $\int_1^\infty \frac{du}{u+u^2}$
18. $\int_1^\infty \frac{d\theta}{\sqrt{\theta^2+1}}$ $\frac{1}{\theta}$ D
19. $\int_2^\infty \frac{d\theta}{\sqrt{\theta^3+1}}$ $\frac{1}{\theta^{1.5}}$ C

$\int^\infty \frac{1}{x^p}dx =$ Conv if $p>1$, div if $p\le 1$