http://www.youtube.com/watch?v=0QQcj_tLIYo

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TIME TO TALK ABOUT INFINITY

SOME QUESTIONS:

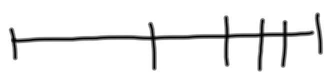
What is infinity?

Are all infinity's the same, or equal?

What do we mean when we say let n→∞?

What would it mean to add up infinitely many terms of an infinite geometric series, like:

$1 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots$



What should $\int_1^{\infty}\frac{1}{x^2}dx$ mean? $1 = \frac{2}{3}+\frac{1}{3}$

List three things you know about working with infinity (infinite series, limits at infinity, whatever)

1) $\infty - \infty =$ any # (depends) $\infty + 1 = \infty$

2) "Olive Garden breadsticks are infinite" $(-\infty, 1]$

3) There's $+\infty, -\infty$ ∞ can be different sizes

Find out three new facts about infinity from whatever sources you like (textbook, online, etc.):

1) ∞ is greater than all real #'s Symbol is called a lemniscate.

2) $(0^{-2} = \infty)$? $0 \cdot \infty$ is "not a real #"

3) The earliest time using ∞ is 450 BCE. There are #'s bigger than infinity.

What are three questions you have about infinity?

1) Can one infinity be bigger than another?

2) Anything w/ series

3) What's the difference between "potential ∞" and "actual ∞"?

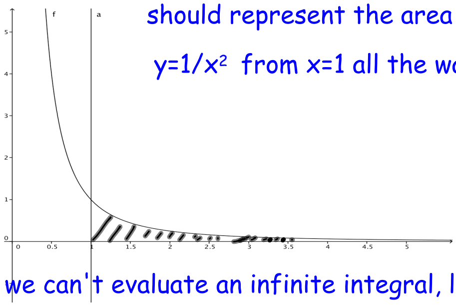
4) Is there a # beyond ∞?

5) How can you find an integral with a limit of ∞?

6) Can we do calculations with ∞?

We call integrals like this improper integrals:

$\int_1^\infty \frac{1}{x^2}dx$ should represent the area under the curve $y=1/x^2$ from $x=1$ all the way to the right



Since we can't evaluate an infinite integral, let's evaluate finite integrals, and see what happens as we keep letting the upper limit of our integral increase:

$$\int_1^\infty \frac{1}{x^2}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^2}dx$$
$$= \lim_{t\to\infty} \left(-\frac{1}{x}\right)\Big]_1^t$$
$$= \lim_{t\to\infty} \left(-\frac{1}{t} - \left(-\frac{1}{1}\right)\right)$$
$$= \lim_{t\to\infty} \left(-\frac{1}{t} + 1\right)$$
$$= 1$$

Now let's try: $\int_1^\infty \frac{1}{x}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}dx$

$$= \lim_{t\to\infty} \ln x\Big]_1^t$$
$$= \lim_{t\to\infty} (\ln t - \ln 1)$$
$$= \lim_{t\to\infty} \ln t = \infty$$

$$\int_1^\infty \frac{1}{x}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}dx$$

What about: $\int_{-\infty}^\infty \frac{1}{1+x^2}dx =$

$$\int_{-\infty}^\infty \frac{1}{1+x^2}dx$$

$$L = \lim_{t\to\infty}\int_a^t f(x)dx$$

So, if the limits of the integral are ∞ or $-\infty$, we will look at the limit as $t\to\infty$

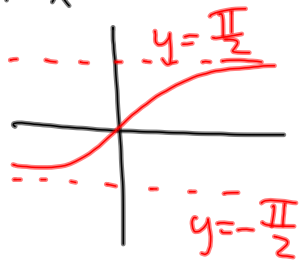
Improper Integrals

TERMINOLOGY: If $L=\lim_{t\to\infty}\int_a^t f(x)dx$ exists and is finite, we say that the integral $\int_a^\infty f(x)dx$ converges to L. Otherwise, we say the the integral diverges.

$$\int_a^\infty f(x)dx$$

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx =$$

$$\int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx =$$



$$\lim_{t\to-\infty} \int_{t}^{0} \frac{1}{1+x^2}\,dx =$$

$$\lim_{t\to-\infty} \arctan x \Big]_t^0 =$$

$$\lim_{t\to-\infty} \arctan 0 - \arctan t =$$

$$\lim_{t\to-\infty} 0 - \arctan t = \frac{\pi}{2}$$

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We can also have integrals that are improper because the integrand itself goes to ∞ over the interval we're integrating.

For example: $\int_0^1 \frac{1}{x^2}dx = \lim_{t\to 0}\int_t^1 \frac{1}{x^2}dx$

Diverges

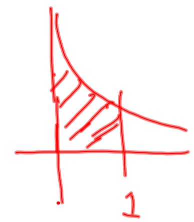
$$= \lim_{t\to 0} \left(-\frac{1}{x}\right)\Big]_t^1$$

$$= \lim_{t\to 0}\left(-\frac{1}{1} - \left(-\frac{1}{t}\right)\right)$$

$$= \infty$$

$$\int x^{-\frac{1}{2}}dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} = 2\sqrt{x}$$

$$\int_0^1 \frac{1}{\sqrt{x}}dx = \lim_{t\to 0^+}\int_t^1 \frac{1}{\sqrt{x}}dx$$



$$= \lim_{t\to 0^+} 2\sqrt{x}\Big]_t^1$$

$$= \lim_{t\to 0^+}\left(2\sqrt{1} - 2\sqrt{t}\right)$$

$$= 2$$

$$\int_1^\infty \frac{1}{\sqrt{x}}dx = \lim_{t\to\infty}\int_1^t \frac{1}{\sqrt{x}}dx$$

Diverges

$$= \lim_{t\to\infty} 2\sqrt{x}\Big]_1^t$$

$$= \lim_{t\to\infty}\left(2\sqrt{t} - 2\sqrt{1}\right)$$

$$= \infty$$

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For example: $\int_0^1 \frac{1}{x^2}dx = \lim_{t\to 0}\int_t^1 \frac{1}{x^2}dx$

=

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{t\to 0^+}\int_t^1 \frac{1}{x^2}\,dx$$

$$= \lim_{t\to 0^+} -\frac{1}{x}\Big]_t^1$$

diverges

$$= \lim_{t\to 0^+}\left(-\frac{1}{1}-\left(-\frac{1}{t}\right)\right)$$

$$= \infty$$

$\int_0^1 \frac{1}{\sqrt{x}}dx$

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$