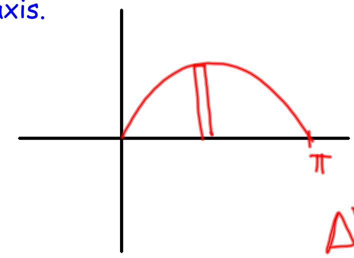
HW: Packet: Page 4 #1 -4, 7 - 10

DO NOW:

Find the volume of the following solid:
It lies between planes perpendicular to the x-axis at x=0 and x=π. The cross sections perpendicular to the x-axis on this interval are quarter circles whose radius lies between the curve y=sin x and the x-axis and whose "center" is on the x-axis.



$A = \frac{\pi r^2}{4}$

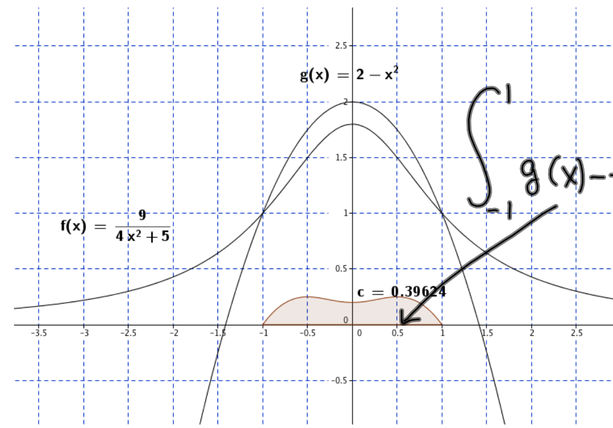
$\Delta V = \frac{\pi r^2}{4} \cdot \Delta x$

$= \frac{\pi y^2}{4} \Delta x$

$= \frac{\pi (\sin^2 x)}{4} \Delta x$

$V = \frac{\pi}{4} \int_0^{\pi} \sin^2 x \, dx$

$= 1.234$



Problem #13
from Page 395

$\int_{-1}^{1} g(x) - f(x) \, dx =$ 0.39624

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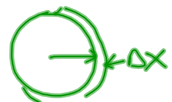
$d = \sec x - \tan x$

Δx

Z

$\Delta V = \pi r^2 \Delta x$

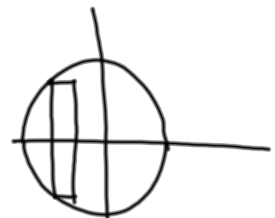


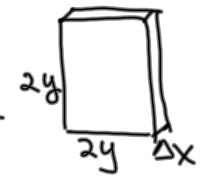
$\Delta V = \pi \left(\frac{\sec x - \tan x}{2}\right)^2 \Delta x$

$V = \frac{\pi}{4}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\sec x - \tan x)^2\, dx$

Write, but do not evaluate, an integral

#5





$y^2 = 1 - x^2$

$\Delta V = 4y^2 \Delta x = 4(1 - x^2)\Delta x$

$V = 2\int_0^1 4(1 - x^2)\, dx = \int_{-1}^{1} 4(1 - x^2)\, dx$

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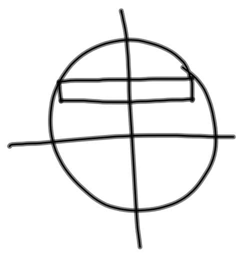

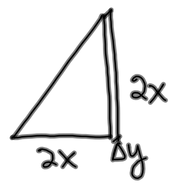

$$\Delta V = \frac{1}{2}\left(4x^2\right)\Delta y$$
$$= 2x^2 \Delta y$$
$$= 2(1-y^2)\Delta y$$

$$V = \int_{-1}^{1} 2(1-y^2)\,dy = 2\int_{0}^{1} 2(1-y^2)\,dy$$

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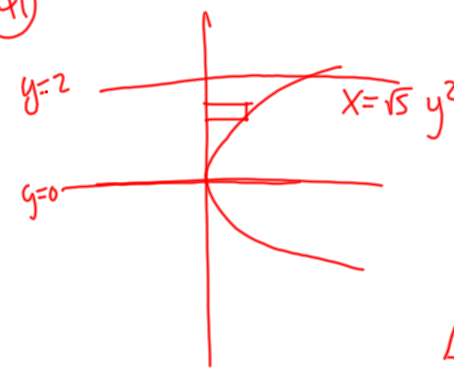
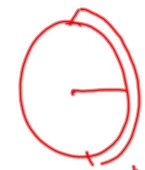




$A = \pi r^2$

$\Delta V = \pi r^2 \Delta y$

$\Delta V = \pi \left(\frac{x}{2}\right)^2 \Delta y$

$\Delta V = \pi \frac{5}{4} y^4 \Delta y$

$$V = \frac{5\pi}{4}\int_{0}^{2} y^4\,dy$$

We will look at Volumes of Solids of Revolution.

We take regions in the plane and revolve them around an axis (in our examples, either vertical or horizontal) to generate a three dimensional shape. All of the cross-sections will be circles (today) or "washers" (next class).

This is really a special case of the Volumes of Known Cross Section that we looked at yesterday.

We can take a look at some of these.

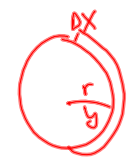
http://mathdemos.org/mathdemos/diskmethod/diskmethodgallery.html

http://www.shodor.org/interactivate/activities/FunctionRevolution/

Some examples:

1. Take the region bounded by $y=x^2$, $y=0$ and $x=1$. Revolve this around the x-axis.

What does a typical cross section look like?



What is $\Delta V = \pi r^2 \Delta x = \pi y^2 \Delta x$

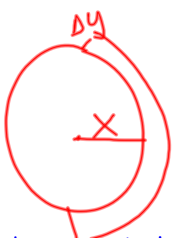
$$= \pi (x^2)^2 \Delta x$$

What is our integral and what is its value?

$$V = \pi \int_0^1 x^4 dx = \left(\pi \frac{x^5}{5}\right)\Big]_0^1 = \frac{\pi}{5}$$

Some examples:

2. Take the region bounded by $y=x^2$, $y=1$ and the y-axis. Revolve this around the y-axis.



What does a typical cross section look like?

$$\Delta V = \pi x^2 \Delta y = \pi y \Delta y$$

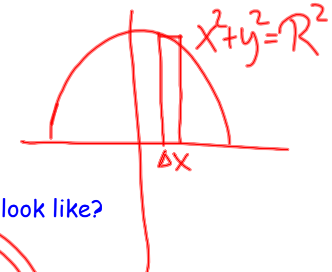
What is ΔV=

$$V = \pi \int_0^1 y\,dy = \left(\tfrac{1}{2}y^2\right)\Big|_0^1 \pi = \frac{\pi}{2}$$

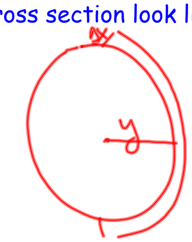
What is our integral

3. Use our method to find the volume of a sphere of radius R.

What is our region?



What does a typical cross section look like?



What is ΔV=

What is our integral

$$\Delta V = \pi y^2 \Delta x = \pi(R^2 - x^2)\Delta x$$

$$V = \pi \int_{-R}^{R} R^2 - x^2\,dx$$

$$V = 2\pi \int_0^R (R^2 - x^2)\,dx$$

$$2\pi\left(R^2 x - \frac{x^3}{3}\right)\Big|_0^R =$$

$$2\pi\left(\left(R^3 - \frac{R^3}{3}\right) - 0\right) = \frac{4}{3}\pi R^3$$