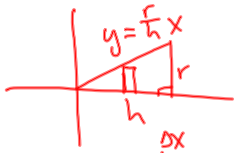
HW: Volume Packet Page 4 #11 - 25 (odd), 55
Checkin Tuesday on Area Between Curves and Volumes

DO NOW:

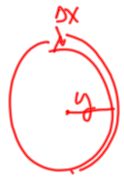
Could someone put up answers to the even HW problems.

Let's use our method to find the volume of a cone of height h and radius r.

What is our region?



What does a typical cross section look like?



$$\Delta V=\pi y^2\Delta x$$
$$=\pi\left(\frac{r}{h}x\right)^2\Delta x$$

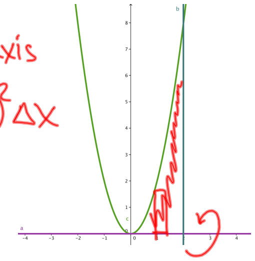
What is ΔV=

What is our integral $V=\pi\int_0^h\left(\frac{r}{h}x\right)^2dx$

$$=\pi\int_0^h\frac{r^2}{h^2}x^2\,dx$$
$$=\frac{\pi r^2}{h^2}\int_0^h x^2dx$$
$$=\frac{\pi r^2}{h^2}\,\frac{1}{3}x^3\Big]_0^h$$
$$=\frac{\pi r^2}{h^2}\cdot\frac{1}{3}h^3$$
$$=\frac{1}{3}\pi r^2h$$

Let's look at problem #12 part b

$y=2x^2$, $y=0$, $x=2$ around x-axis



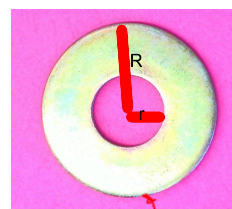
$$\Delta V=\pi y^2\Delta x=\pi(2x^2)^2\Delta x$$
$$V=\pi\int_0^2 4x^4dx$$

Next Question:
How do we deal with regions that are revolved around an axis that is not the boundary of the region?

Look at Volumes of Revolution

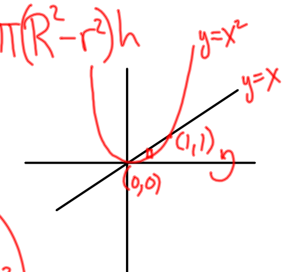
Region bounded by: $y=x$, and $y=x^2$ revolved around the x-axis

What do our cross-sections look like?

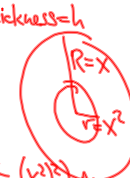


WASHERS: How do we find the volume of a washer?

$V=\pi(R^2-r^2)h$



What is ΔV? Thickness=h



$$\Delta V=\pi\left(x^2-(x^2)^2\right)\Delta x$$

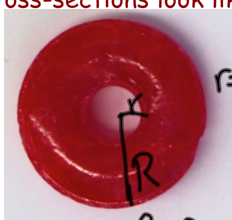
What is our integral:

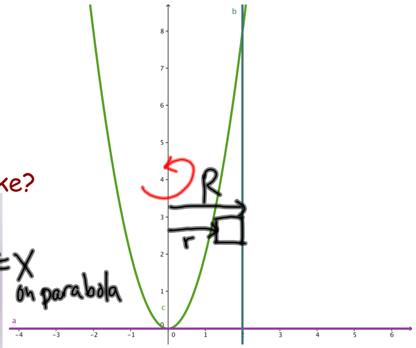
$$V=\pi\int_0^1 x^2-x^4\,dx$$

Let's look at 12a)

$y=2x^2$, $y=0$, $x=2$ revolved around the y-axis $\quad x^2=\frac{1}{2}y$

What do our cross-sections look like?





$r = x$ on parabola

$R=2$

What is ΔV?

$$\Delta V = \pi\left(2^2 - x^2\right)\Delta y$$

$$\Delta V = \pi\left(2^2 - \left(\tfrac{1}{2}y\right)\right)\Delta y$$

What is our integral:

$$V = \pi\int_0^8 4 - \tfrac{1}{2}y\,dy$$

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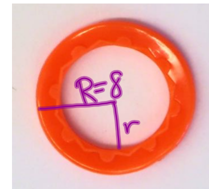
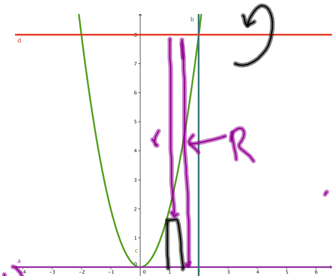
We can also revolve around other horizontal and vertical lines

12c) Around y=8





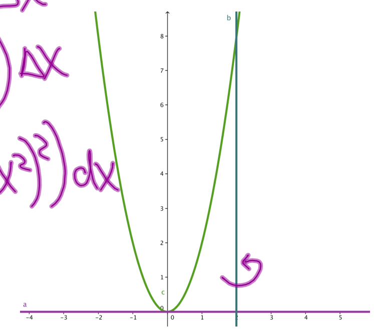
$r = 8 - y$ (parabola)

$$\Delta V = \pi\left(8^2 - (8-y)^2\right)\Delta x$$

$$= \pi\left(8^2 - (8-2x^2)^2\right)\Delta x$$

$$V = \pi\int_0^2 \left(8^2 - (8-2x^2)^2\right)dx$$

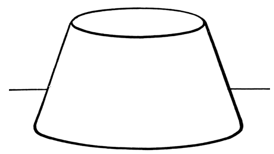
12d) Around x=2



Try #16 $y=6-2x-x^2$, $y=x+6$

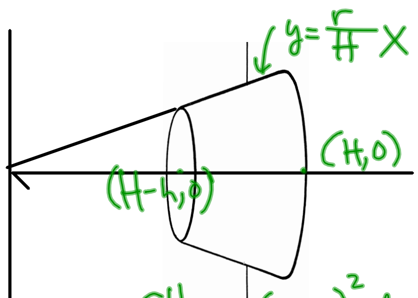
a) Around the x-axis

b) Around the line y=3



Volume Packet, page 5 #51:
A cone with a base of radius r and height H is cut by a plane parallel to and h units above the base.
Find the volume of the solid (frustrum of a cone) below the plane.

HINT BELOW:



$$V=\pi\int_{H-h}^{H}\left(\frac{r}{H}x\right)^2dx$$

$$=\frac{\pi r^2}{H^2}\int_{H-h}^{H}x^2dx$$