HW: Due Next Tuesday: Page 395 #3 - 15 (odd)

Do Now:

Complete the Area Between Two Curves worksheet.

Try not to worry about having perfect rectangles. Just sketch them as well as you can.

Math 504

AREA BETWEEN TWO CURVES

Here is the graph of f(x) = sin (x) and g(x) = cos (x)

1) Find the x coordinates of the two intersection points.

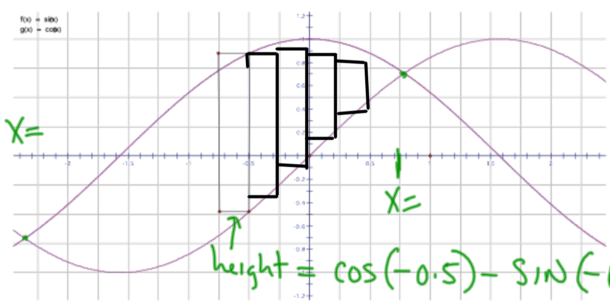
$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}$

$-\frac{3\pi}{4}$

2) For the region bounded by the two curves, draw "right hand" rectangles that approximate the area between the two curves. Make the ones that are not at the "ends" have a width of 0.25 (I have drawn the one that goes from -0.75 to -0.5 for you).



height = $\cos(-0.5) - \sin(-0.5)$

3. How do you calculate the heights of each of the rectangles?

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APPLICATIONS OF INTEGRATION

We will use the same strategy to learn how to use definite integrals to calculate quantities other than areas under a curve:

1) Divide our interval up into subintervals.
2) On each subinterval, approximate the quantity.**
3) Create a sum which is a Riemann sum for a continuous function on our interval.
4) As we let the subintervals go to zero, our Riemann sum approaches a limiting integral.

** We need to make sure that this approximation is "good enough."

How does this work for Area Between Two Curves:

Our Riemann sum is $\sum_{k=1}^{n}\left(f(x_k)-g(x_k)\right)\Delta x_k$

In the limit, this becomes $\int_{x=a}^{x=b}\left(f(x)-g(x)\right)dx$

So, the area between f(x)=sin x and g(x)= cos x that we are looking for is given by:

$$\int_{x=-3\pi/4}^{\pi/4} \cos x - \sin x \, dx =$$

and the value is:

This applet lets us look at what we've done.

~~http://www.calculusapplets.com/area.html~~

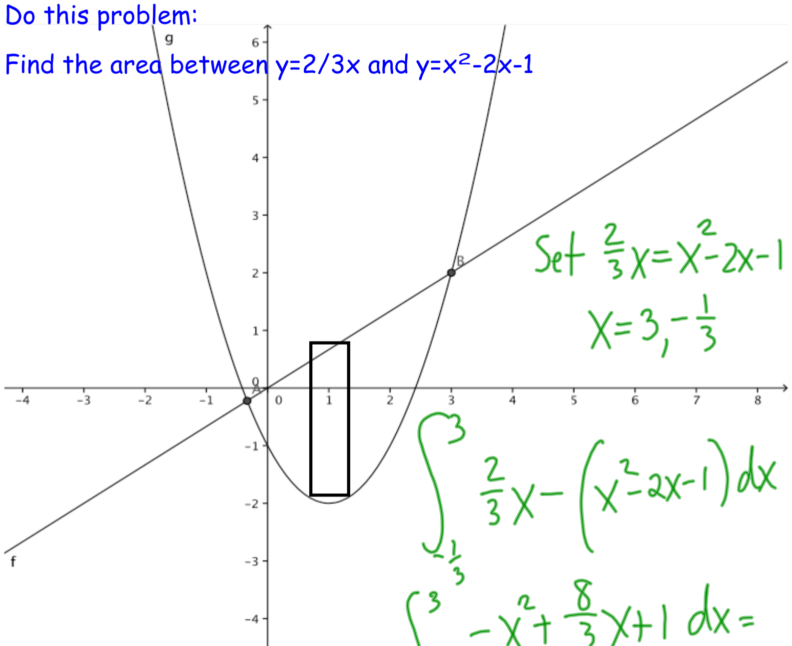
$$\sin x + \cos x \Big]_{-\frac{3\pi}{4}}^{\frac{\pi}{4}} =$$

$$\left(\sin\left(\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{4}\right)\right)-\left(\sin\left(-\frac{3\pi}{4}\right)+\cos\left(-\frac{3\pi}{4}\right)\right)-$$

$$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{2}}{2}+-\frac{\sqrt{2}}{2}\right)=2\sqrt{2}$$

Do this problem:

Find the area between $y=2/3x$ and $y=x^2-2x-1$



Set $\frac{2}{3}x = x^2 - 2x - 1$

$x = 3, -\frac{1}{3}$

$$\int_{-\frac{1}{3}}^{3} \frac{2}{3}x - \left(x^2 - 2x - 1\right) dx$$

$$\int_{-\frac{1}{3}}^{3} -x^2 + \frac{8}{3}x + 1 \, dx =$$

$$\left. -\frac{1}{3}x^3 + \frac{4}{3}x^2 + x \right]_{-\frac{1}{3}}^{3}$$

$$\left(-\frac{1}{3}(3)^3 + \frac{4}{3}(3)^2 + 3\right) - \left(-\frac{1}{3}\left(-\frac{1}{3}\right)^3 + \frac{4}{3}\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)\right)$$

On the AP exam, you can stop here

We can do the same process on functions defined in terms of y.

Example: Find the area bounded by the curve $x=1-y^2$ and the lines $x=y+2$, $y=1$ and $y=-1$

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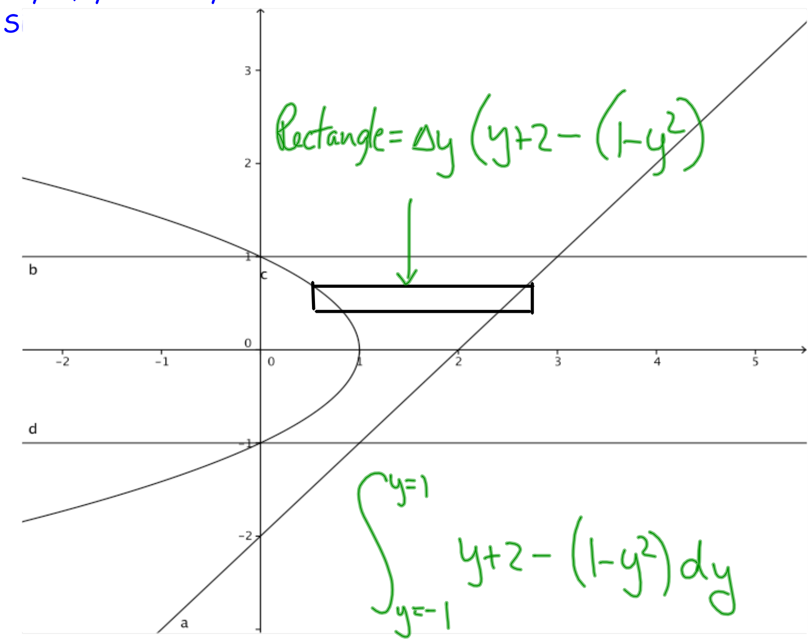

Rectangle $= \Delta y \left(y+2-\left(1-y^2\right)\right)$

$$\int_{y=-1}^{y=1} y+2-\left(1-y^2\right) dy$$

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