HW: Page 499 #1 - 6 [integration by parts]
Page 412 #53, 59, 61, 70-75 [u-substitution]
Unit Test is Friday, Jan. 13, 2017

DO NOW:

Go over HW.

Questions?

35 $\int x^3 (x^4-1)^2 dx =$

$u = x^4 - 1 \quad du = 4x^3 dx$

$\frac{1}{4} du = x^3 dx$

$\frac{1}{4}\int u^2 du = \frac{1}{4}\left(\frac{1}{3}u^3\right) + C$

$= \frac{1}{12}(x^4-1)^3 + C$

Our next strategy: A different kind of product problem:

Let's recall the product rule:

$(f(x)\cdot g(x))' = f'(x)g(x) + f(x)g'(x)$

Running this backwards, we get:

$\int \underbrace{(f(x)\cdot g(x))'}_{} dx = \int f'(x)g(x)dx + \int f(x)g'(x)dx$

$f(x)\cdot g(x)$

Reorganizing terms we get

$$\int f(x)\cdot g'(x)dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)dx$$

This is called Integration by Parts

$$\int f(x)\cdot g'(x)dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)dx$$

How does this help us: We start with one product and end up with another? The trick is that sometimes the new product will be a function that we can find an antiderivative for. For integration by parts to work, we need to choose f(x) and g(x) carefully.

Example:

$$\int xe^x dx = x\cdot e^x - \int 1\cdot e^x dx$$

$$\boxed{\begin{array}{ll} f(x)=x & g'(x)=e^x \\ f'(x)=1 & g(x)=e^x \end{array}}$$

$$= xe^x - e^x + C$$

$$\int f(x)\cdot g'(x)dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)dx$$

Try: $\int x\cos(2x)dx = \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)dx$

$$\boxed{\begin{array}{ll} f(x)=x & g'(x)=\cos(2x) \\ f'(x)=1 & g(x)=\frac{1}{2}\sin(2x) \end{array}}$$

$$= \frac{1}{2}x\sin(2x) - \frac{1}{2}\left(-\frac{1}{2}\cos(2x)\right) + C$$

$$= \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$$

Integration by Parts January 4, 2017

The standard notation for integration by parts is:

$$\int u\,dv = uv - \int v\,du$$

$$u = f(x) \qquad dv = g'(x)dx$$

Let's use this notation in an example:

$$\int x^2\ln(x)dx = \frac{1}{3}x^3\ln(x) - \int \frac{x^3}{3}\cdot\frac{1}{x}dx$$

$$\boxed{\begin{array}{ll} u=\ln(x) & \int dv = \int x^2 dx \\ du = \frac{1}{x}dx & v = \frac{x^3}{3} \end{array}}$$

$$= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^2 dx$$

$$= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\cdot\frac{x^3}{3} + C$$

$$\int \arctan x \, dx = x\arctan x - \int \frac{x}{1+x^2}\,dx$$

$$\boxed{\begin{array}{ll} u = \arctan x & dv = 1 \cdot dx \\ du = \frac{1}{1+x^2}\,dx & v = x \end{array}}$$

from u-sub notes

$$= x\arctan x - \tfrac{1}{2}\ln(1+x^2) + C$$

$$= x\arctan x - \ln\sqrt{1+x^2} + C$$