HW: Page 411 #1-7 (all), 9,10, 23 -27 (odd)

DO NOW:

Go over HW, any questions?

18 $$\int \frac{dx}{x^2+a^2} \stackrel{?}{=} \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C$$

$$\frac{d}{dx}\left(\frac{1}{a}\arctan\left(\frac{x}{a}\right)\right)=\frac{1}{a}\,\frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{a}$$

$$=\frac{1}{a^2\left(1+\frac{x^2}{a^2}\right)}=\frac{1}{a^2+x^2}$$

11 $$\int \frac{(1-x)^3}{\sqrt{x}}\,dx=\int x^{-\frac{1}{2}}\left(1+3x^2-3x-x^3\right)dx$$

21 $$\int \frac{dx}{1-x^2}=\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|+C$$

$$\frac{d}{dx}\left(\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\right)=$$

$$\frac{d}{dx}\left(\frac{1}{2}\left[\ln|1+x|-\ln|1-x|\right]\right)=$$

$$\frac{1}{2}\left(\frac{1}{1+x}\cdot 1-\frac{1}{1-x}\cdot(-1)\right)=$$

$$\frac{1}{2}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)=$$

$$\frac{1}{2}\left(\frac{1-x+1+x}{1-x^2}\right)=$$

$$\frac{1}{2}\left(\frac{2}{1-x^2}\right)=\frac{1}{1-x^2}$$

#22 from the HW

$$\frac{d}{dx}\left(x\arcsin x+\sqrt{1-x^2}\right)$$

$$\underline{1\cdot\arcsin x}+\frac{x}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{1-x^2}}\cdot -2x$$

$$\arcsin x+\frac{x}{\sqrt{1-x^2}}-\frac{x}{\sqrt{1-x^2}}$$

Our Next Step in learning how to find antiderivatives:
Take the chain rule and run it backwards!

Chain Rule says: $\left(f(g(x))\right)'=f'(g(x))\cdot g'(x)$

Run it backwards: $\int f'(g(x))\cdot g'(x)dx=f(g(x))$

This method is called "u-substitution"

How do we use this?
If we have an integrand that is a product; look to see if one term is a composition and the other term is the derivative of the inner function.

$$\int 2x\cos(x^2)dx=\int f'(g(x))\cdot g'(x)\,dx=f(g(x))+C$$
$$=\sin(x^2)+C$$

$g(x)=x^2 \qquad f'(g(x))=\cos(x^2)$

$g'(x)=2x \qquad f(x)=\sin(x)$

Name:____________________ Date:____________________

504: BC Calculus (AP) u-substitution introduction

What do you want in the box so that you can integrate?

1) $\int \cos\left(x^3\right)\boxed{3x^2}dx$

2) $\int \sqrt{\tan x}\,\boxed{\sec^2 x}\,dx$

3) $\int \sec^2\sqrt{x}\,\boxed{\frac{1}{2\sqrt{x}}}dx$

4) $\int \sin\left(\cos x\right)\boxed{-\sin x}dx$

5) $\int \sqrt{\left(e^{5x}\right)}\;\boxed{5e^{5x}}dx$

6) $\int \sec^2\left(\tan^{-1}x\right)\boxed{\frac{1}{1+x^2}}dx$

$$\int f'(g(x))\cdot g'(x)dx$$

u-substitution December 21, 2016

Why is this method called u-substitution?

$\Delta u = \frac{\Delta u}{\Delta x}\cdot\Delta x$ $\quad \frac{du}{dx}$ deriv of u wrt x

We need to define a differential $\quad du = \frac{du}{dx}dx$

In spite of what anyone (including yourself) might tell you, this is NOT fraction cancellation. We make a definition for the differential du that makes it look like fraction rules work.

We will call our inner function u and then rewrite our integral in terms of u.

So in the integral $\int f'(g(x))g'(x)dx$

we replace g(x) with u so $g'(x)dx = \frac{du}{dx}dx$ which is what we just defined as du.

When we're done we need to "translate back" to x.

Redoing the previous problem: $\int 2x\cos\left(x^2\right)dx$

$u = x^2 \qquad du = 2xdx$ Definition

$$\int 2x\cos\left(x^2\right)dx = \int \cos(u)du$$
$$= \sin(u) + C$$
$$= \sin\left(x^2\right) + C$$

$$\int \sqrt{\tan x}\, \sec^2 x\, dx =$$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x\, dx \end{aligned}}$$

$$\int \sqrt{u}\, du =$$

$$\int u^{1/2}\, du =$$

$$\frac{2}{3} u^{3/2} + C =$$

$$\frac{2}{3}(\tan x)^{3/2} + C$$

$$\int \sec^2 \sqrt{x}\, \frac{1}{2\sqrt{x}}\, dx =$$

$$\boxed{u = x^{\frac{1}{2}} \quad du = \frac{1}{2} x^{-\frac{1}{2}} dx}$$

$$\int \sec^2 u\, du =$$

$$\tan u + C$$

$$\tan \sqrt{x} + C$$

Some more examples:
(MAKE SURE TO PUT THESE IN YOUR NOTES

$$\int \frac{1}{1+x^2}dx = \arctan(x) + C$$

$$\int \frac{2x}{1+x^2}dx = \int \frac{du}{u} = \int \frac{1}{u}\,du = \ln|u| + C$$

$u = 1+x^2$

$du = 2x\,dx$

$$= \ln|1+x^2| + C$$

$$= \ln(1+x^2) + C$$

$$\int \frac{\ln x}{x}dx = \int u\,du = \frac{u^2}{2} + C$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{x}{1+x^4}dx$$

$u = x^2$

$du = 2x\,dx$ $\quad \frac{1}{2}du = x\,dx$

$$\int \frac{x}{1+x^4}dx = \int \frac{1}{2}\frac{du}{1+u^2} = \frac{1}{2}\arctan(u) + C$$

$$= \frac{1}{2}\arctan(x^2) + C$$