HW: AP Problem tub of water

DO NOW:

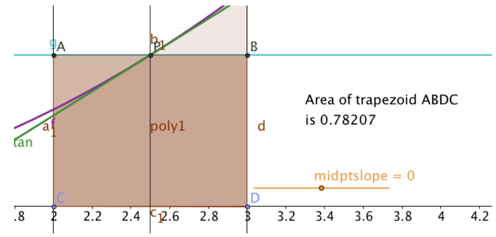
Go over HW with your groups. Any questions?

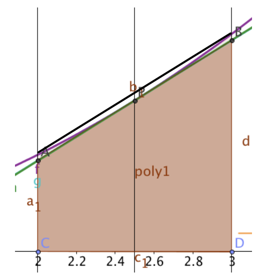
When you calculate an approximation, we multiply units, eg.

$\frac{\text{miles}}{\text{hr}} \cdot \text{hrs}$

$= \text{miles}$

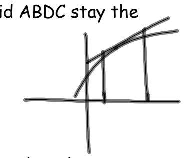
This file (midpt approx graphic.ggb) helps us to understand the midpoint approximation in a different way.





Area of trapezoid ABDC
is 0.78207

Why does the area of trapezoid ABDC stay the same?



How can we describe this trapezoid that has the same area as the midpoint sum rectangle?

The tangezoid© is the trapezoid whose upper side is the tangent at the midpt.

©Alex Shames

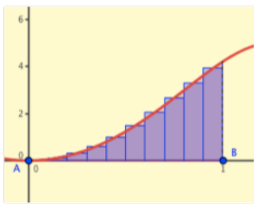
Midpt sum rectangle has the same area as the tangezoid.

This can help us decide if we have an over or underestimate.

Looking at Approximations: Good, Better, Best

Here are screen shots of the Geogebra file riemann4.ggb with the function:

$$g(x) = 5\sin(x^2)$$



Left Hand Sum	1.3455
Right Hand Sum	1.7662
MidPoint Sum	1.5491
Trapezoid Sum	1.5559
Simpson's Approx	1.5513
Integral Value	1.5513

Intervals: n = 10 From a = 0 to b = 1

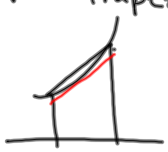
1. Which approximation is pictured here? midpt
2. Which sums are below the actual value? left, midpt

   Which sums are above the actual value? right hand, trapezoid
3. Which sums are farther from the actual value? left, right

   Which sums are closer to the actual value? trapezoid, midpt
4. Explain how you could figure out the answers to 2 and 3 without using the actual calculations, just by looking at the graph.

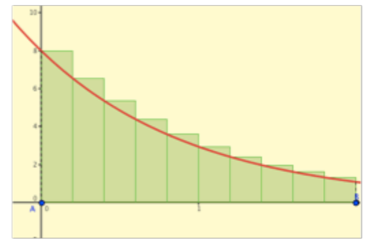
If f(x) is monotone (increasing all the time OR decreasing all the time) then left and right are always upper and lower sums

Concavity (if it stays the same) tells us which is above, which is below for trapezoid and midpt



Here are screen shots of the Geogebra file riemann4.ggb with the function:

$$g(x) = 8e^{-x}$$



Left Hand Sum	7.6321
Right Hand Sum	6.2486
MidPoint Sum	6.9058
Trapezoid Sum	6.9404
Simpson's Approx	6.9173
Integral Value	6.9173

Intervals: n = 10 From a = 0 to b = 2

1. Which approximation is pictured here? left
2. Which sums are below the actual value? right midpt

   Which sums are above the actual value? left trap
3. Which sums are farther from the actual value? left, right

   Which sums are closer to the actual value? trap, midpt
4. Explain how you could figure out the answers to 2 and 3 without using the actual calculations, just by looking at the graph.

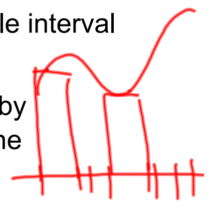
Can you generalize your answers for these two graphs to figure out how you would know the answers to questions 2 and 3 for a function just by looking at the graph. Justify your answer.

What is a Riemann sum?

A more general case of our right, left and midpoint sums.

Any "rectangular" approximating sum is a Riemann sum.

The bases for the rectangles do NOT need to be of equal width. They do need to cover the whole interval without overlapping.



It is okay to find the height of the rectangles by evaluating the function at random points in the intervals.

As the width of the rectangles become smaller, the Riemann sums will approach the definite integral.

We say (and write) that the definite integral is the limit of the Riemann sums. If the intervals have equal width, then the limit is taken as the number of intervals goes to infinity.

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\cdot \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

GREATEST HITS

For the test you should be able to:

1) Define the definite integral in terms of area and know the integral properties that follow from this.

2) Calculate the exact value of a definite integral using area/geometry, properties of integrals and knowledge of the behavior of functions (i.e. symmetry).

3) Use appropriate notation for writing definite integrals

4) Determine when you can calculate a definite integral (understanding the impact of the continuity of a function),

5) Calculate the average value of a function (understanding the connection to area) $\frac{\int_a^b f(x)dx}{b-a}$

6) Know the Fundamental Theorem of Calculus (FTC) and use it to evaluate definite integrals using the ANTIDERIVATIVE.

7) Use the FTC in problems involving graphs of integral defined functions to **justify** behavior of functions (values, extrema, concavity, etc.) $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

8) Use the FTC to solve problems involving rates IN and rates OUT.

9) Find values for approximations: Left, Right, Midpoint and Trapezoid.

10) In special cases we've looked at, determine how good various estimates are in comparison with each other, and decide if they are overestimates or underestimates.

11) CAREFUL:
   a) Use an estimation sign for all estimates.
   b) When asked to explain the meaning of a mathematical expression in the context of a problem be very explicit and precise about WHAT, WHEN, and UNITS. Class Problem
   c) Make sure that you answer all parts of each question.
   d) Make sure you are using the requested approximation.

You will have your calculator.
I will clear calculators before and after the test.
Answers correct to 3 decimal places, unless instructed otherwise.
Write 4 places to be safe.

The Definite Integral