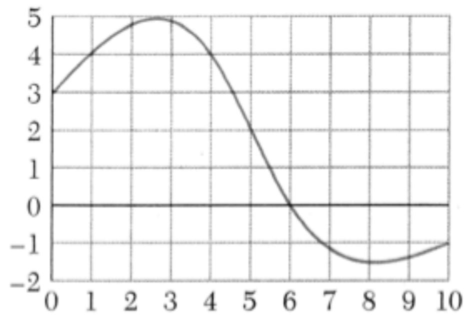
Page 354 Problem #10 Ostebee Zorn

Graph of h



Let h be the function shown graphically in Exercise 8 and suppose that $h(t)$ represents the *eastward* velocity (in meters/second) of an object moving along an east-west axis at time t seconds. ~~Estimate each of the following quantities (use appropriate units!)~~ and interpret in the language of velocity, distance, etc.

mph

miles/hour^2

~~mph^2~~

(a) $\int_0^{10} h(t)\,dt$ = displacement in meters eastward from start pt from t=0 seconds to t=10 seconds

(b) $\frac{1}{10}\int_0^{10} h(t)\,dt$ = average eastward velocity in m/s from t=0 secs to t=10 secs.

(c) $\frac{1}{10}(h(10) - h(0))$ = average eastward acceleration in m/s^2 from t=0 secs to t=10 secs

(d) $|h(8)|$ = speed at t=8 seconds

(e) $\int_0^{10} |h(t)|\,dt$ = total distance traveled in meters from t=0 secs to t=10 secs

(f) $\frac{1}{10}\int_0^{10} |h(t)|\,dt$ = average speed in m/s from t=0 secs to t=10 secs

interpret the **meaning** (remember tha this means, with unit, on a certain domain/interval)

2010 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

1 : answer

(c) $h(0) = 0$

For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0.$

For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6).$

For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7).$

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$

$3 : \begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$

$h(t) + \int_7^t g(s)\,ds = \boxed{h(t)} \cancel{+h(7)}$

(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$