HW: Two AP Problems, Graph problem and Camp Newton Problem

Do Now:

Consider these integrals and decide what they **mean** when f(t)=velocity function in units of meters per second.

1) $\int_a^b f(t)\,dt$ = displacement in meters from time t=a seconds to t=b seconds

2) $\int_a^b |f(t)|\,dt$ = distance traveled in meters from time t=a seconds to t=b seconds

Note: More generally, we can think about integrals as "accumulation of rate". This is the focus of our work today.

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

IN $+$ **No calculator is allowed for these problems.**   OUT $-$

$+$

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

   (a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

   (b) How many gallons of water are in the tank at time $t = 3$ minutes?

   (c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

   ? (d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

a) $\int_0^3 \sqrt{t+1}\,dt = 4\frac{2}{3}$ gallons

b) Initial amount + Amount added − leaked out amount

$30 + 8\cdot 3 - \int_0^3 \sqrt{t+1}\,dt = 49\frac{1}{3}$

c) $A(t) = 30 + 8t - \int_0^t \sqrt{x+1}\,dx$

$= 30 + \int_0^t 8 - \sqrt{x+1}\,dx$

d) $A'(t) = 8 - \sqrt{t+1}$

Rate in = Rate out

$A'(t) = 0$ when $t = 63$

So $A'(t)$ changes $+$ to $-$ at $t = 63$, this gives a max

$A(3) = 30 + 8(3) + \int_0^3 \sqrt{3+1}\,dt$

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$\frac{dL}{dt} = \sqrt{t+1}$; $\quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$

$L(0) = 0$; $\quad C = -\frac{2}{3}$

$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}$; $\quad L(3) = \frac{14}{3}$

Method 1:

$3\begin{cases} 2: \text{definite integral} \\ \quad 1: \text{limits} \\ \quad 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

– or –

Method 2:

$3\begin{cases} 1: \text{antiderivative with } C \\ 1: \text{solves for } C \text{ using } L(0) = 0 \\ 1: \text{answer} \end{cases}$

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

1 : answer

(c) Method 1:

$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx$

$\quad = 30 + 8t - \int_0^t \sqrt{x+1}\,dx$

– or –

Method 2:

$\frac{dA}{dt} = 8 - \sqrt{t+1}$

$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$

$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C$; $\quad C = \frac{92}{3}$

$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$

Method 1:

$2\begin{cases} 1: 30 + 8t \\ 1: -\int_0^t \sqrt{x+1}\,dx \end{cases}$

– or –

Method 2:

$2\begin{cases} 1: \text{antiderivative with } C \\ 1: \text{answer} \end{cases}$

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

$3\begin{cases} 1: \text{sets } A'(t) = 0 \\ 1: \text{solves for } t \\ 1: \text{justification} \end{cases}$

by EVT, consider endpoints $t = 0, 120$

$A(0)$

$A(63)$

$A(120)$