Another way to interpret the FTC:
If f(t) is a nice function on the interval [a,b] and
If we think of f'(t) as a rate of accumulation, then
we know from the FTC:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

What does this mean: Integrating a rate over time t=a to t=b gives change in the amount over that time interval

$$\underbrace{f(b)}_{\text{Final amount}} = \underbrace{f(a)}_{\text{Initial quantity}} + \underbrace{\int_a^b f'(x)dx}_{\text{Amount "added"}}$$

What about something like this...

Don't need this

$$\frac{d}{dx}\left[\int_1^x \sin t\, dt\right] = \frac{d}{dx}\left(-\cos t\Big]_1^x\right) = \frac{d}{dx}\left(-\cos x + \cos 1\right) =$$

$$\sin x$$

$$\frac{d}{dx}\left[\int_1^x e^{\cos^2 t}\, dt\right] = e^{\cos^2 x}$$

What will this look like on the AP Exam?

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

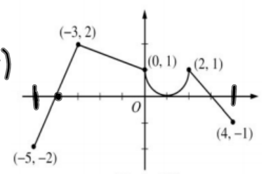
$g'(x) = f(x)$

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.



Graph of f

$x = -3, 1, 2$

Function value $= y$

Check endpts

−5	
−4	
4	

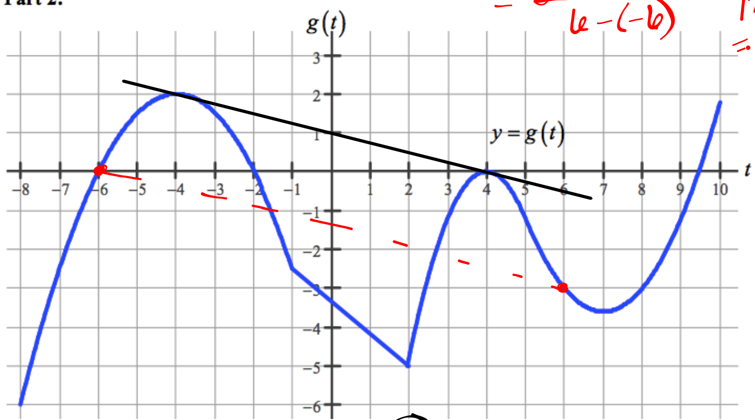
(a) $g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$ $g'(0) = f(0) = 1$	$2: \begin{cases} 1: g(0) \\ 1: g'(0) \end{cases}$
(b) g has a relative maximum at $x = 3$. This is the only x-value where $g' = f$ changes from positive to negative.	$2: \begin{cases} 1: x = 3 \\ 1: \text{justification} \end{cases}$
(c) The only x-value where f (= g') changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints. $g(-5) = 0$ $g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$ $g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13-\pi}{2}$ So the absolute minimum value of g is -1.	$3: \begin{cases} 1: \text{identifies } x = -4 \text{ as a candidate} \\ 1: g(-4) = -1 \\ 1: \text{justification and answer} \end{cases}$
(d) $x = -3, 1, 2$	2 : correct values ⟨−1⟩ each missing or extra value

POI when $g' = f$ changes from increasing to decreasing or vice versa.

HOMEWORK

$$= \frac{\boxed{g(6) - g(-6)}}{6-(-6)} = \frac{-3-0}{12} = -\frac{1}{4}$$

Part 2:



The figure above shows the graph of a function g continuous on the closed interval $-8 \le t \le 10$. The graph has horizontal tangent lines at $x = -4$, $x = 4$, and $x = 7$. Let $f(x) = \int_{-2}^{x} g(t)\,dt$.

1. On what intervals is the function $f(x)$ increasing? Justify your answer.
2. On what intervals is the function $f(x)$ decreasing? Justify your answer.
3. On what intervals is the function $f(x)$ concave upwards? Justify your answer.
4. On what intervals is the function $f(x)$ concave downwards? Justify your answer.
5. What are the x-coordinates of any local maximum values of $f(x)$. Justify your answer.
6. What are the x-coordinates of any local minimum values of $f(x)$. Justify your answer.
7. What are the x-coordinates of any points of inflection of $f(x)$. Justify your answer.
8. Consider the interval $-6 \le t \le 6$:
   a. What is the average rate of change of $g(t)$ over this interval?
   b. Does the Mean Value Theorem guarantee a value of c, in this interval, such that $g'(t)$ is equal to this average rate of change? Justify your answer?
   c. Is there such a value of c? Explain.