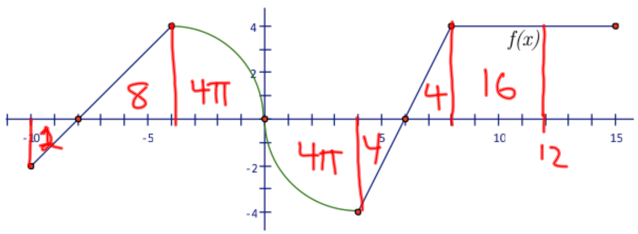
2. The function f shown below consists of line segments and quarter circles.



(a) $\int_0^6 f(x)\,dx =$

$\int_0^4 f(x)dx + \int_4^6 f(x)dx =$

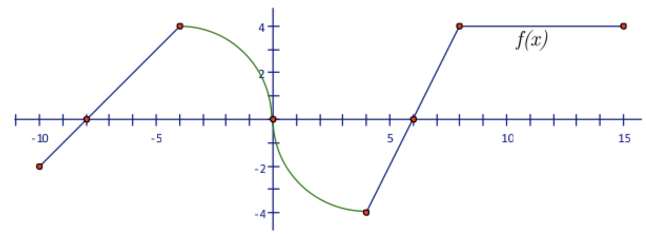
-4 $\quad -4\pi$

(b) $\int_{-8}^0 f(x)\,dx$

$\int_{-4}^0 f(x)dx + \int_{-8}^{-4} f(x)dx$

$4\pi + 8$

2. The function f shown below consists of line segments and quarter circles.



(c) $\int_{12}^8 f(x)\,dx =$

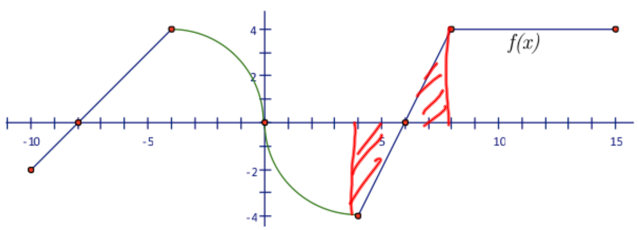
$-\int_8^{12} f(x)dx = -16$

(d) $\int_6^{10} \left(f(x)+3\right) dx =$

$\int_6^{10} f(x)dx + \int_6^{10} 3dx =$

$12 + 12 = 24$

2. The function f shown below consists of line segments and quarter circles.

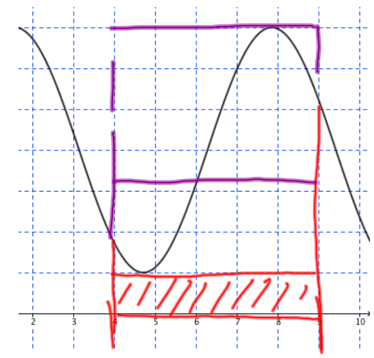


(e) Determine all values of k if $\int_0^k f(x)\,dx = 0$

$k=0$

$k=\pi+8$

Given y=3sin(x)+4 on the interval [4,9], what is the smallest possible (reasonable) approximation for the area bounded by the curve and the x-axis?Largest possible approximation?



$$5 < \int_4^9 (3\sin x+4)\,dx < 35$$

Given y=3sin(x)+4 on the interval [4,9], what is the smallest possible (reasonable) approximation for the area bounded by the curve and the x-axis?Largest possible approximation?

$5 \le \int_4^9 f(x)\,dx \le 35$

(1·5) (7·5)

$Y_1 = 3\sin(x)+4$

Let's look at how to have the calculator find the area of this region with good precision.

Vars, Y-Vars, Function Math, fnInt(   )

$\int_4^9 (Y_1(X))\,dX$ = fnInt(function, variable, a, b)

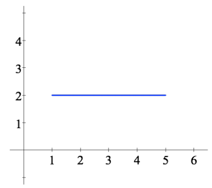
$Y_1(x)$, X, 4, 9

20.772

Work through this packet with your new group. See if you can figure out how we should define the average value of a function on an interval.

Average Value of a Function **Name**________________________

1. Consider the function $f(x)=2$ for $1 \le x \le 5$.



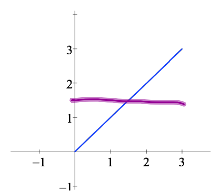
a. What are the y-coordinates of the points of this function?

2

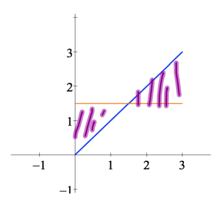
b. What is their average?

2

2. a. Now consider the function $y=x$ for $0 \le x \le 3$. The y-coordinates go quite evenly from 0 to 3. What is the average of all the y-coordinates? ______ Explain why you think this is so?

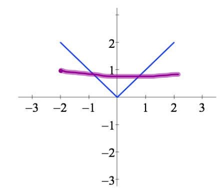


b. The graph shows the function and a horizontal segment at $y=\frac{3}{2}$. Does it seem reasonable that the y-coordinates of the points on $y=x$ above this segment will "balance" the y-coordinates below the segment?



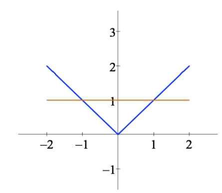
c. Find the area of the region between $y=x$ and the x-axis between 0 and 3. How does it compare to the area between the horizontal segment and the x-axis?

3. Now consider the function $y=|x|$ for $-2 \le x \le 2$.



a. What appears to be the average of the y-coordinates of this function?

b. In the next drawing there is a horizontal segment drawn at $y=1$. Does this segment seem to "balance" the y-coordinates of the function above and below it?

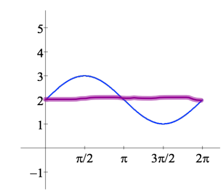


c. How do the areas of the regions between the function and the x-axis and between the horizontal segment and the x-axis compare?

4. Next consider $y=2+\sin(x)$ for $0 \le x \le 2\pi$.

a. What seems to be the average value of all the y-coordinates? _______ Draw a horizontal line at this value.

b. Do the y-coordinates above and below this value seem to balance out?



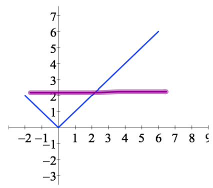
c. Find the area between $y=2+\sin(x)$ for $0 \le x \le 2\pi$ and the x-axis? (Show your work)

d. Is your answer to c the same as the area under the average value line on this interval?

5. (Be careful!) This time the function is $y = |x|$ for $-1 \le x \le 6$

a. What appears to be the average value? _________

b. Draw a horizontal segment at the average value. Does the amount of the graph above and below seem to balance? How about the areas; are they the same?



c. If things are not working as before, try using the area between the function and the *x*-axis to find the average value.

$$2.643 = \frac{\text{Area}}{\text{interval}}$$

6. Consider the function $y = x + 2\cos(x)$ for $0 \le x \le 5$.

a. Use the area idea to find the average value.

$$2.1164 = \frac{\int_0^5 (x + 2\cos x)\,dx}{5}$$

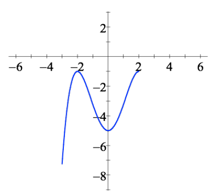
b. Draw a horizontal line as before. Does the horizontal line look like it's in the right place?

7. Consider the function $y = -\frac{1}{4}x^4 + 2x^2 - 5$ for $-3 \le x \le 2$.

a. Use the area of the region between the graph and the *x*-axis to find the average value.

b. Draw a horizontal line as above. Does the horizontal line look like it's in the right place?

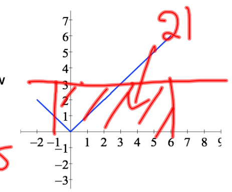
c. Your answer should be negative. Why?



5. (Be careful!) This time the function is $y = |x|$ for $-1 \le x \le 6$

a. What appears to be the average value? _________

b. Draw a horizontal segment at the average value. Does the amount of the graph above and below seem to balance? How about the areas; are they the same?



$$\int_{-1}^{6} |x|\,dx = 18.5$$

c. If things are not working as before, try using the area between the function and the *x*-axis to find the average value.

$\int_{-1}^{6} |x|\,dx = 18.5 = 7$(avg value on $[-1, 6]$)

6. Consider the function $y = x + 2\cos(x)$ for $0 \le x \le 5$.

a. Use the area idea to find the average value.

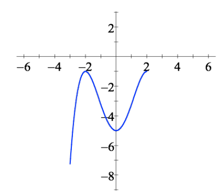
b. Draw a horizontal line as before. Does the horizontal line look like it's in the right place?

7. Consider the function $y = -\frac{1}{4}x^4 + 2x^2 - 5$ for $-3 \le x \le 2$.

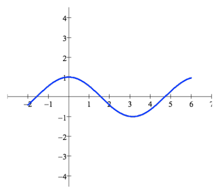
a. Use the area of the region between the graph and the *x*-axis to find the average value.

b. Draw a horizontal line as above. Does the horizontal line look like it's in the right place?

c. Your answer should be negative. Why?



8. Consider the function $y = \cos(x)$ for $-2 \le x \le 6$



a. Use the area idea to find the average value.

b. Draw a horizontal line as above. Does the horizontal line look like it's in the right place?

c. Part of the region here has a "negative area." How does that affect finding the average value?

9. Summarize your results with a formula that will give you the average value (average of the *y*-coordinates) of a function $f(x)$ on an interval $a \le x \le b$.

Average value = $\dfrac{\int_a^b f(x)dx}{b-a}$