HW: Page 353 #6,13,15,17, #21-24
Please complete the written reflection on first quarter by next Tuesday.

DO NOW: Please go to the assignments page at my website. Go to today's assignment and click on the link to complete the survey about the beginning of the year in this course. Thanks.

What we noticed:

Sometimes we got negative area

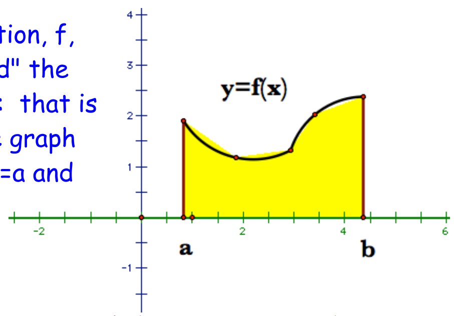
"You keep the negative sign when you're in quadrant II & IV

The area function $A_2(x)$ was an antiderivative of $g(x)$

Starting our next BIG topic:

WHAT IS AREA?

For any "reasonable" function, f, we want to be able to "find" the area of the shaded region: that is the region bounded by the graph of f, the x-axis, the line x=a and the line x=b.



To start with we will define a symbol that represents this quantity:
For any function defined on $a \le x \le b$ we write:

limits of integration: upper b, lower a

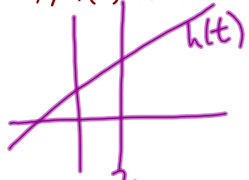
$\int_a^b f$ or $\int_a^b f(x)\,dx$

$f(x)$: integrand; dx: variable of integration

to represent the **signed** area bounded by x=a, x=b, y=f(x) and the x-axis.

What is signed area?
We count area above the x-axis as positive
and area below the x-axis as negative.

We read $\int_a^b f$ or $\int_a^b f(x)\,dx$ as "the integral of f from a to b."

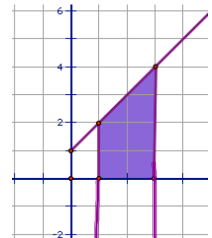


$\int_2^1 h(t)\,dt < 0$

??

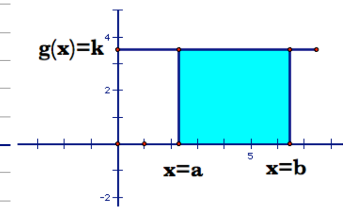
We can easily name this quantity, but CAN WE CALCULATE IT?

If we choose our functions carefully, we can do this for some functions. Let's try a few. Use geometry to calculate the following integrals.



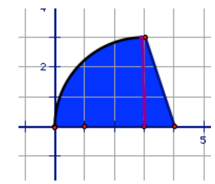
$f(x) = x + 1$

$\int_1^3 (x+1)\,dx = 6$



$g(x) = k$

$\int_a^b g(x)\,dx = k(b-a)$



h(x)= a quarter-circle followed by a segment

$\int_0^4 h(x)\,dx = \frac{9\pi}{4} + 1.5$

$\int_1^3 -(x+1)\,dx = -6$

Definite Integral $\int_a^b f(x)\,dx$

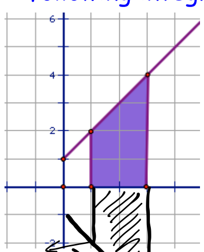
$\int_3^1 (x+1)\,dx = -6$ It's a number

$\int_3^1 -(x+1)\,dx = 6$

We can easily name this quantity, but CAN WE CALCULATE IT?

If we choose our functions carefully, we can do this for some functions. Let's try a few. Use geometry to calculate the following integrals.

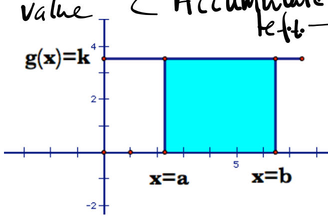
integral value > 0 { function is positive
Accumulate in positive direction left → right



$y = -(x+1)$

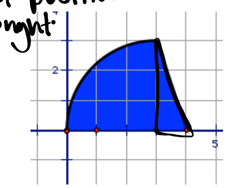
$f(x) = x + 1$

$\int_1^3 (x+1)dx = 6$



$g(x) = k$

$\int_a^b g(x)dx = (b-a)k$



h(x)= a quarter-circle followed by a segment

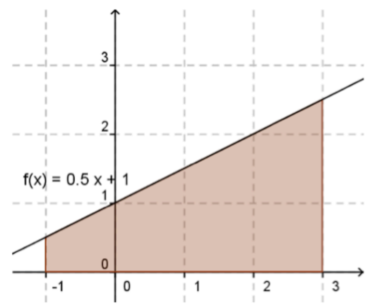
$\int_0^4 h(x)dx = \frac{9\pi}{4} + \frac{3}{2}$

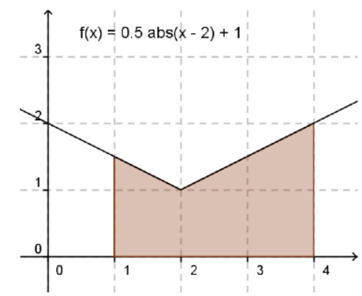
$\int_3^1 (x+1)dx = -6$

$\int_1^3 -(x+1)dx = -6$

$\int_3^1 -(x+1)dx = 6$

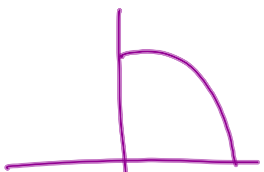
1. Consider the shaded regions below. For each, (a) write a definite integral expression that equals the area of the shaded region and (b) evaluate the integral using formulas from geometry.



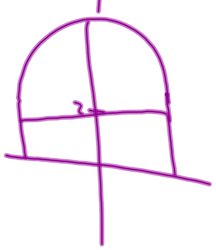


$\int_{-1}^3 (0.5x+1)dx = 6$

$\int_1^4 (0.5|x-2|+1)dx = 4\frac{1}{4}$



$\Rightarrow \int_0^4 \sqrt{16-x^2}\,dx = 4\pi$



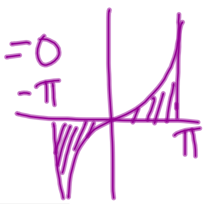
$\int_{-4}^{4} 2+\sqrt{16-x^2}\,dx \quad 16+8\pi$

Properties of Integrals

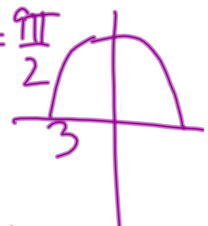
$\int_a^a f(x)\,dx = 0$	*Area of nothing*
$\int_a^b c\cdot dx = (b-a)\cdot c$	*Rectangle shape*
$\int_a^b \big(f(x)+g(x)\big)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$	*Whole is equal to the sum of its parts I*
$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$	*Whole is equal to the sum of its parts II*
$\int_a^b k\cdot f(x)\,dx = k\cdot \int_a^b f(x)\,dx$	*Vertical stretch*
$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	*Opposites*

1. Evaluate the following definite integrals and explain your reasoning.

(a) $\int_{-\pi}^{\pi} x^3\,dx = 0$

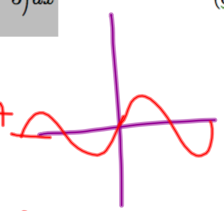


(b) $\int_{-3}^{3} \sqrt{9-x^2}\,dx = \frac{9\pi}{2}$



(c) $\int_{-2\pi}^{2\pi} \left(\sin x + 5\right) dx$

$\underbrace{\int_{-2\pi}^{2\pi} \sin x\,dx}_{0} + \int_{-2\pi}^{2\pi} 5\,dx = 20\pi$



(d) $\int_{-2}^{2} x \cos x\,dx = 0$

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