

HW: Finish up Open Response AP questions. We will go over the Multiple Choice problems on Wednesday.

The test is on Wednesday, Nov. 30, 2016

Prove: Differentiability Implies Continuity

ie. If  $f'(a)$  exists, then  $f(x)$  is continuous at  $x=a$ .

Where are we starting?

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \text{ exists}$$

Where are we trying to get to?

$$\lim_{x \rightarrow a} f(x) = f(a) \Leftrightarrow \lim_{x \rightarrow a} (f(x) - f(a)) = 0$$

Proof:

We are given that

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a) \text{ exists}$$

We need to show that

$$\lim_{x \rightarrow a} f(x) - f(a) = 0$$

Which is equivalent to

So let's consider:

$$\lim_{x \rightarrow a} f(x) - f(a) =$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \cdot (x - a) =$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \rightarrow a} (x - a) =$$

$$f'(a) \cdot 0 = 0$$

## Greatest Hits

### 1) Related Rates

Relate, then rate, do the steps  
Write a sentence for your answer  
Implicit differentiation with respect to  $t$ .

### 2) Linearization

Know the process  
Do the work carefully

### 3) Theorems

IVT  
EVT  
MVT  
Diff  $\Rightarrow$  Continuity

Know the hypotheses  
For MC, read each choice carefully

Format: 5 or 6 Multiple Choice  
2 Short Answer  
1 Open Response

### Suggested Review:

Multiple Choice on Theorems and Related Rates  
 AP Problems on Mean Value Theorem and Related Rates  
 Matching Problems on Related Rates

### Using Derivatives

### #3 Examples involving MVT

a)  $f(x)$  is differentiable on  $[0, 7]$   
 because differentiability implies continuity  
 $f(x)$  is also continuous on  $[0, 7]$ .

We have  $f(4) = 9$  and  $f(6) = 9$ ,  
 by MVT, there's a  $c \in (4, 6)$  so that

$$f'(c) = \frac{f(6) - f(4)}{6 - 4} = \frac{0}{2} = 0$$

b) Similar argument for hypotheses

Use  $c_1$  from above as well as  
 $c_2$  from  $[0, 2]$  with MVT.

MVT Free Response Questions

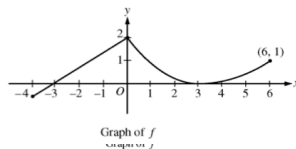
Math 504 – Calculus BC

**AP EXAM FREE-RESPONSE QUESTIONS - THE MEAN VALUE THEOREM**  
**What do MVT questions look like on the AP Free-Response Questions?**

On some questions, the student is explicitly asked to use the Mean Value Theorem. On others, the use of the MVT is implied. The conditions of continuity on a specific closed interval and differentiability on the open interval must be acknowledged in order for the student to apply the MVT to justify the MVT conclusion. Students need to carefully read the stem of each question looking for the terms continuous and differentiable (or twice-differentiable).

**2009 SCORING GUIDELINES (Form B)**

**Question 3**



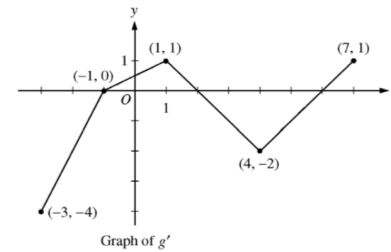
A continuous function  $f$  is defined on the closed interval  $-4 \leq x \leq 6$ . The graph of  $f$  consists of a line segment and a curve that is tangent to the  $x$ -axis at  $x = 3$ , as shown in the figure above. On the interval  $0 < x < 6$ , the function  $f$  is twice differentiable, with  $f''(x) > 0$ .

- (a) Is  $f$  differentiable at  $x = 0$ ? Use the definition of the derivative with one-sided limits to justify your answer.
- (b) For how many values of  $a$ ,  $-4 \leq a < 6$ , is the average rate of change of  $f$  on the interval  $[a, 6]$  equal to 0? Give a reason for your answer.
- (c) Is there a value of  $a$ ,  $-4 \leq a < 6$ , for which the Mean Value Theorem, applied to the interval  $[a, 6]$ , guarantees a value  $c$ ,  $a < c < 6$ , at which  $f'(c) = \frac{1}{3}$ ? Justify your answer.

MVT Free Response Questions

Math 504 – Calculus BC

**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**



5. Let  $g$  be a continuous function with  $g(2) = 5$ . The graph of the piecewise-linear function  $g'$ , the derivative of  $g$ , is shown above for  $-3 \leq x \leq 7$ .
  - (a) Find the  $x$ -coordinate of all points of inflection of the graph of  $y = g(x)$  for  $-3 < x < 7$ . Justify your answer.

- (b) For the function  $g$  on the interval  $[-3, 7]$ , where do you think the global maximum value will be? Explain your answer.

Your answer here will not match the one in the rubric, which includes material we haven't covered yet.

- (d) Find the average rate of change of  $g'(x)$  on the interval  $-3 \leq x \leq 7$ . Does the Mean Value Theorem applied on the interval  $-3 \leq x \leq 7$  guarantee a value of  $c$ , for  $-3 < c < 7$ , such that  $g''(c)$  is equal to this average rate of change? Why or why not?