HW: Page 338 #1, 3, 4, 6, 9
MC on Related Rates and Theorems is due next Tuesday
Test is Wednesday, 11/30/16

DO NOW:

Use the IVT to show: If f is continuous on [0,1] and has a range contained in [0,1] then there exists a c in [0,1] such that f(c)=c.

Note: c is called a fixed point of f.

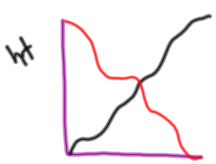
HINT: Consider the function g(x)= f(x)-x, look at the largest and smallest possible values for g(x) at the endpoints of the interval.

(And what about the monk?)

Need to show $g(x)=0$
$\Rightarrow$ f(x)=x for that x

$g(0)=f(0)-0=f(0)$ $0\le f(0)\le 1$
If $f(0)=0$ we're done, otherwise $g(0)>0$
$g(1)=f(1)-1$ so $-1\le g(1)\le 0$
If $g(1)=0$, then $f(1)=1$, we're done.
Otherwise $g(1)<0$
Then since $g(0)>0$ and $g(1)<0$, there's a place where $g(x)=0$.

$u(t)-d(t)$



We will do #1-4 as a large group and then you will explore #5-8

The Mean Value Theorem

There are some concepts in the calculus that are very simple and yet very important. Sometimes their importance overshadows their simplicity. The concept called the Mean Value Theorem (MVT) is often one of these. So let's start with its simplest form.

You've all traveled somewhere, near or far, in a car. How do you find your average speed on the trip? Easy: you take the total distance you traveled and divide it by the total time of the trip.

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

1. First question for you: Was there a time during the trip when you were actually traveling at the average speed? When the car's odometer actually showed that that your speed at that instant was that average speed? Explain your answer.

If your speed is constant, it's the average all the time.
Sometime you're above, sometime you're below
IVT says you have to pass through the average.

We will do #1-4 as a large group and then you will explore #5-8

The Mean Value Theorem

There are some concepts in the calculus that are very simple and yet very important. Sometimes their importance overshadows their simplicity. The concept called the Mean Value Theorem (MVT) is often one of these. So let's start with its simplest form.

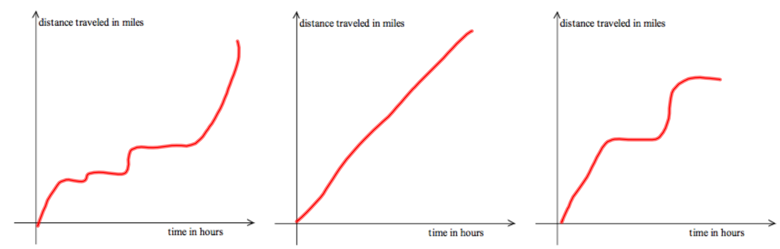
You've all traveled somewhere, near or far, in a car. How do you find your average speed on the trip? Easy: you take the total distance you traveled and divide it by the total time of the trip.

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

1. First question for you: Was there a time during the trip when you were actually traveling at the average speed? When the car's odometer actually showed that that your speed at that instant was that average speed? Explain your answer.

If your speed is constant, then it's your average.
If not, at times you're above average, to average out you have to be below, therefore, by IVT have to equal average at some time.

2. Next: Suppose you drew a graph of you trip. The time since you departed on the *t*-axis and the distance you traveled on the *y*-axis. Keep in mind that the distance you traveled is not the same as the distance from your starting point; in fact, your trip may have ended at the same place it started. Sketch several versions what this graph might look like.



3. In calculus terms, are your graphs continuous? Why?

Yes, Reality, no teleports

4. In calculus terms are your graphs differentiable? Why?

Yes- the derivative represents the speed of the car

5. Draw the segment between the endpoints of your graphs. What is the slope of this segment in terms of the distance traveled and the elapsed time? Include units of measure.

slope = average speed for trip

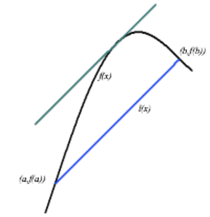
6. The slope of the line is your average velocity; your average rate of change of distance with respect to time. At any point on the graph the slope of the tangent line is the instantaneous rate of change of distance with respect to time. Sketch on your graphs above in a different color the tangent line at all points where the tangent appears to be parallel to the segment between the endpoints.

The graphs you drew should have been *continuous* (you were always some distance from your start even if you stopped) and *differentiable* (assuming you didn't stop instantly by hitting tree or something). Your graph should have been non-decreasing (increasing or horizontal). Continuity and differentiability are important, non-decreasing is not. Any graph that is continuous (on a closed interval) and differentiable (on the open interval) has the property that at least one point the tangent line will be parallel to the chord between the endpoints. *There is a point where the instantaneous rate of change is equal to the average rate of change.*

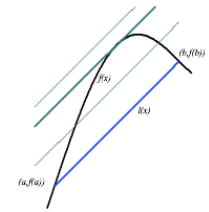
Mean Value Theorem and what it impllies November 17, 2016

This result, in slightly different wording, is called the Mean Value Theorem.

Geometric interpretation: There is at least one point where the tangent line to a graph of a function that meets the hypotheses of the MVT is parallel to the chord between the endpoints. Graphically, this should be obvious.



This leads to a graphical "proof." Picture the chord between the endpoints moving up or down parallel to its original position. Eventually all or part of the chord will lie outside of the region enclosed by the graph and the original position of the chord. Just as it leaves it is tangent to the graph. This is where the tangent line is parallel to the chord between the endpoints.



TODAY'S BIG THEOREM:

MEAN VALUE THEOREM

If f is continuous on [a,b] and differentiable on (a,b) then there exists some c in (a,b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

This means:

1) Instantaneous velocity = average velocity on an interval, at least once

2) Tangent to the curve (at least at one point) is parallel to the secant connecting (a,f(a)) and (b,f(b)).

MVT tangent secant.ggb

What follows from the Mean Value Theorem

1) If f'(x)=0 for all x in an interval, then f is constant on the interval, ie show for any a, b in the interval f(a)=f(b).

Where are we starting?

$f'(x)=0$ for all x interval

Where do we want to get to?

For any $a\neq b$ $f(a)=f(b)$

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$0=\frac{f(b)-f(a)}{b-a}\Rightarrow f(b)=f(a)$$

2) If F'(x)=G'(x) on an interval I, then F(x)=G(x)+C on I.

Where are we starting?

$F'(x)=G'(x)$ on I

Let $H(x)=F(x)-G(x)$

So $H'(x)=F'(x)-G'(x)=0$ on I

Where do we want to get to?

$F(x)=G(x)+C \Leftarrow$ So $H(x)=C$

3) If f'(x)>0 on an interval I, then f(x) is increasing on I; ie if a<b (both in I), then f(a)<f(b) or f(b)-f(a)>0.

Where are we starting?

Where do we want to get to?