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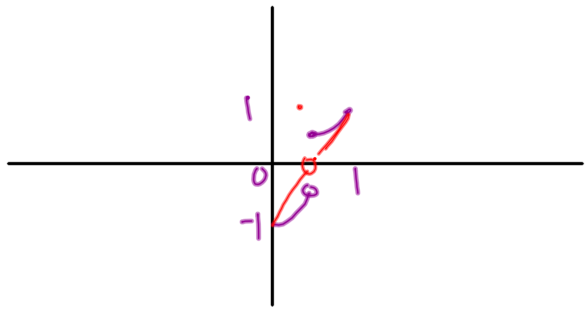
Test on Wednesday, Dec. 1, 2016 on Linearization, Related Rates and Theorems

DO NOW:

Go over HW with your group. Questions?

Try this: Draw the graph of a function, f(x), that is defined for all x in [0,1] such that f(0)=-1 and f(1) =1 and f(x)≠0 for any x in (0,1).



Why is this impossible if f is continuous?

Our first major theorem:

<u>Intermediate Value Theorem</u> (IVT)-
Let f be continuous on the closed, bounded interval [a, b] and let y be any number between f(a) and f(b). Then, for some input c between a and b, f(c) = y.

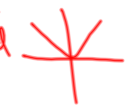
(In other words, you are guaranteed to pass every y-value on your way from f(a) to f(b) if f is continuous.)

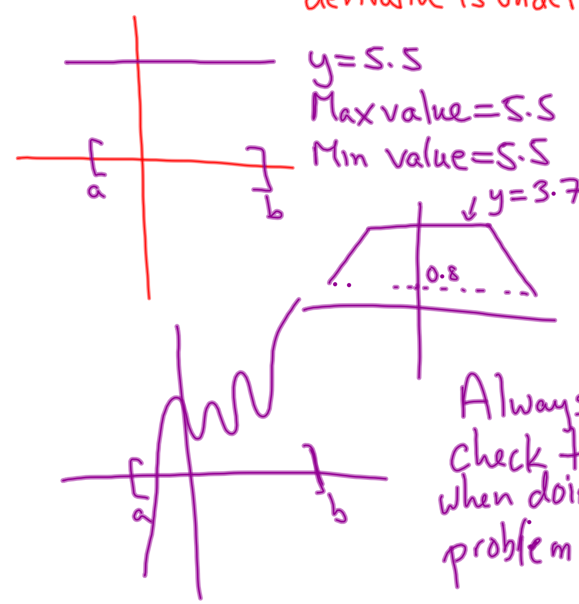
(The IVT says that if f is continuous on [a, b] then the range of f contains not just f(a) and f(b) but everything in between.)

Another Important Theorem: What are the similarities?

<u>Extreme Value Theorem</u> -
Let f be continuous on the closed bounded interval [a, b]. Then f assumes both a maximum value and a minimum value somewhere on [a, b].
(Either at x = a, x = b, or x = c where c is a critical point.)

Critical points = stationary points or derivative is undefined

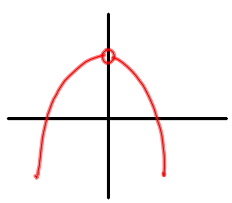




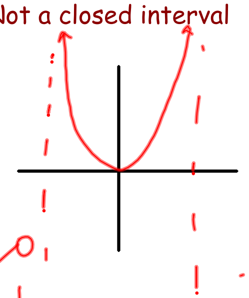
Always need to check the endpts when doing global max/min problem on closed interval

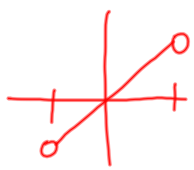
Notice that both theorems require that we have functions that are continuous on closed intervals. For the Extreme Value Theorem, find counterexamples where the conclusion doesn't hold if we don't satisfy one of the hypotheses.

Not continuous

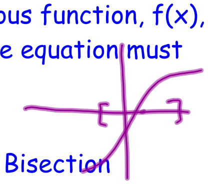


Not a closed interval





A major use of the IVT is to determine where an equation must have roots. If the equation involves a continuous function, f(x), then we know that if f(a)>0 and f(b)<0 then the equation must have a root between a and b.



Our graphing calculators use a version of the Bisection Method to find zeros of functions, by looking for where a function changes sign.

$y = x^2$, no sign change

https://numerical-analysis.uibk.ac.at/bisection

We can use this link to see the Bisection Method in action on the polynomial $P(x)=x^3 -3x^2 +2$

We can easily see that P(2)=-2 and P(4)=18, so there must be a zero somewhere between 2 and 4

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