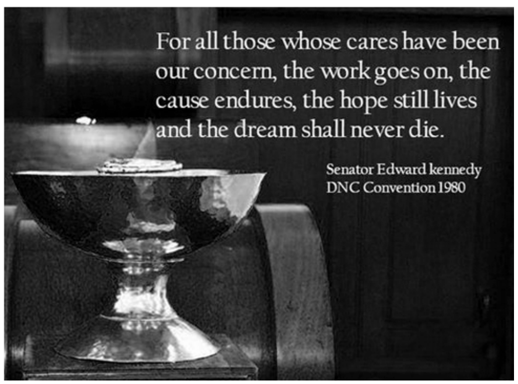
HW: Relate, then Rate packet: Problems #9, 11, 13 (page 251 in packet)

We will spend the next few weeks looking at applications of differentiation. Our first application, Related Rates, uses our new tool, Implicit Differentiation, so we will be practicing that skill while we do these problems.



How does implicit differentiation come in?

Consider a sphere whose radius is changing with respect to time such that *r* is a differentiable function of time. If V= volume of the sphere, find an equation that relates *dV/dt* and *dr/dt*.

$$V = \frac{4}{3}\pi r^3$$
$$V(t) = \frac{4}{3}\pi\left[r(t)\right]^3$$
$$\frac{dV}{dt} = V'(t) = \frac{4}{3}\pi \cdot 3\left(r(t)\right)^2 \cdot r'(t) = 4\pi r^2 \frac{dr}{dt}$$

If the radius, *r*, and the height, *h*, of a cone are differentiable functions of time and V= volume of the cone, find an equation that relates *dV/dt, dr/dt and dh/dt*.

$$V = \frac{1}{3}\pi r^2 h$$
$$V(t) = \frac{1}{3}\pi\left[r(t)\right]^2 \cdot h(t)$$
$$V'(t) = \frac{2}{3}\pi\left(r(t)\right)\cdot r'(t) h(t) + \frac{\pi}{3}\left(r(t)\right)^2 h'(t)$$
$$\frac{dV}{dt} = \frac{2\pi}{3} r h \frac{dr}{dt} + \frac{\pi}{3} r^2 \frac{dh}{dt}$$

Talk about tomorrow

TRY problem #3 on page 251 in the Related Rates packet.

$$V = \pi r^2 h$$

a) $V(t) = \pi r^2 h(t)$

$$\frac{dV}{dt} = V'(t) = \pi r^2 \frac{dh}{dt}$$

b) $V(t) = \pi\left(r(t)\right)^2 h$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt}$$

c) $\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$

In related rates problems, we have variables that are **related**, which are also changing, thus, have **rates** of change. Using the relationship and given enough of the rates of change, we can find other rates of change.

EXAMPLE: (IN THE PACKET) A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 y miles north of the intersection and the car is 0.8 x miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. $\frac{dr}{dt}$ If the cruiser is moving at 60 mph $\frac{dy}{dt}$ at the instant of measurement, what is the speed of the car?

The packet presents a good strategy for solving problems like this. An important part of this strategy is to RELATE, then RATE!

STEPS:

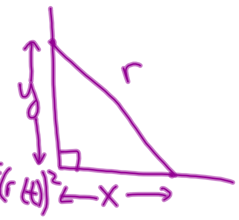
1. Understand the problem: what variables are involved.
2. Develop a mathematical model of the problem: draw a picture.
3. RELATE: Write an equation relating the variable whose rate of change you seek with the variable(s) whose rate of change you know.
4. RATE: Differentiate both sides of the equation implicitly with respect to time *t.*
5. Substitute values for any quantities that depend on time.
6. Interpret the solution.

Follow along with me as we solve the police problem:

(This is in the packet, so you don't need to write everything down.)

Step 1: x = dist of car from corner
y = dist " police " "
r = dist between 2 vehicles

Step 2:



Step 3:

$$x^2+y^2=r^2 \qquad (x(t))^2+(y(t))^2=(r(t))^2$$

Step 4:

$$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=\cancel{2}r\frac{dr}{dt}$$

Step 5: $\frac{dx}{dt}=?$ $\frac{dy}{dt}=-60$ $\frac{dr}{dt}=20$

$x=0.8$ $y=0.6$ $r=1.0$ (Use Pythag)

Step 6:

$$0.8\frac{dx}{dt}+0.6(-60)=(1.0)(20)$$

$$\frac{dx}{dt}=70 \text{ mph}$$