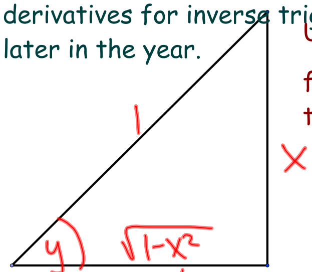
HW: Page 245 #13-28 (all), 29 - 33 (odd) Checkin Tomorrow

Here are three triangles that we will use to help us find derivatives for inverse trig functions. We may use them again later in the year.

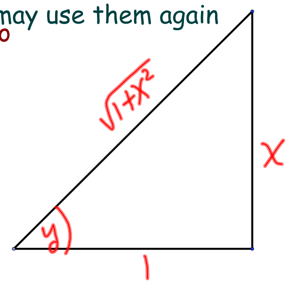
Use the triangle to find $\cos(\sin^{-1}x)$ in terms of x



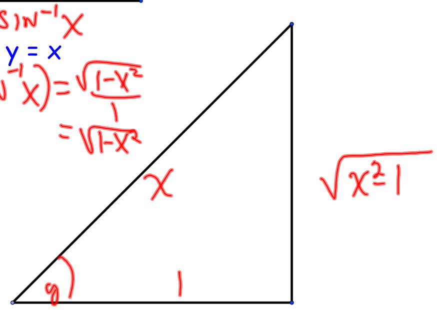
$y = \sin^{-1}x$

sin y = x

$$\cos(\sin^{-1}x) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$



tan y = x

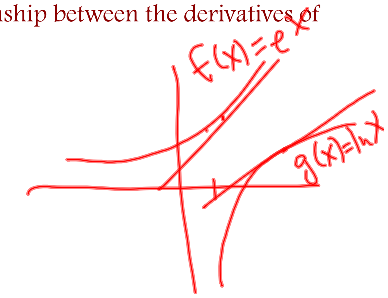


sec y =x

Since our definition of inverses involves the composition, we can use the chain rule to help us see the relationship between the derivatives of a function and its inverse

$$(f \circ g)(x) = f(g(x)) = x$$

$$(f \circ g)'(x) = \boxed{f'(g(x)) \cdot g'(x) = 1}$$



Solving for g'(x), we get

$$g'(x) = \frac{1}{f'(g(x))}$$

If g(a)=b, then

$$g'(a) = \frac{1}{f'(g(a))} = \frac{1}{f'(b)}$$

$$g'(x) = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

To find the derivative of arcsin x, start with
sin(arcsin x)) = x (part of our definition of inverse functions)
Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\sin(\arcsin x) = x)$$

$$\cos(\arcsin x)\frac{d(\arcsin x)}{dx} = 1$$

$$\frac{d(\arcsin x)}{dx} = \frac{1}{\cos(\arcsin x)}$$

How convenient that we know what cos(arcsin x) is!

$$\frac{d(\arcsin x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

A very similar argument would let us show the following:

$$\frac{d(arccos x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

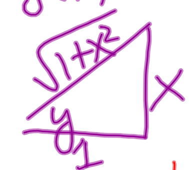
To find the derivative of arctan x, we will use the rule for inverses:

$$g'(x) = \frac{1}{f'(g(x))}$$

$f(x) = \tan x$
$f'(x) = \sec^2 x$
$g(x) = \arctan x$

$$\frac{d(\arctan x)}{dx} = \frac{1}{\sec^2(\arctan x)}$$

$$= \frac{1}{x^2+1}$$



$y = \arctan x$

For inverse secant, we will need to be careful about the "SIGN" of one of the terms in the derivative.

$$\frac{d\left(\sec^{-1}x\right)}{dx}=\frac{1}{\sec\left(\sec^{-1}x\right)\cdot\tan\left(\sec^{-1}x\right)}=\frac{1}{SEC(SEC^{-1}X)TAN(SEC^{-1}X)}$$

$$=\frac{1}{|x|\sqrt{x^2-1}}=\frac{1}{|X|\sqrt{X^2-1}}$$

$$\frac{d(SECX)}{dx}=SECX\,TANX$$

$$\tan(SEC^{-1}X)=\pm\sqrt{X^2-1}$$

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Guess and check antiderivatives:

$$f(x)=\frac{3}{9+x^2}$$

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