Coming Attraction (optional): Preview Proof that the derivative of sin x is cos x ; bottom of page 209 to top of page 211

Due by Friday: Reflections on Unit Test 1b

CHECKIN Friday

DO NOW:

1) Find the following derivatives using the definition of the derivative;

(You may check your answer with the power rule)

f(x)=1/x

$$f'(x) = \lim_{h\to 0} \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \lim_{h\to 0} \frac{x-(x+h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-\cancel{1h}}{\cancel{1h}(x)(x+h)} = -\frac{1}{x^2}$$

g(x)=√x

$$f'(a) = \lim_{x\to a} \frac{(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})}{(x-a)(\sqrt{x}+\sqrt{a})}$$

$$= \lim_{x\to a} \frac{\cancel{x-a} \quad 1}{\cancel{(x-a)}(\sqrt{x}+\sqrt{a})}$$

$$= \frac{1}{2\sqrt{a}} = \frac{1}{2}a^{-\frac{1}{2}}$$

2) Use the power rule to find the second, third, fourth, nth derivatives of f(x)=1/x $f^{(n)}(x) =$

$f(x) = x^{-1}$

$f^{(1)}(x) = -x^{-2}$

$f^{(2)}(x) = 2x^{-3}$

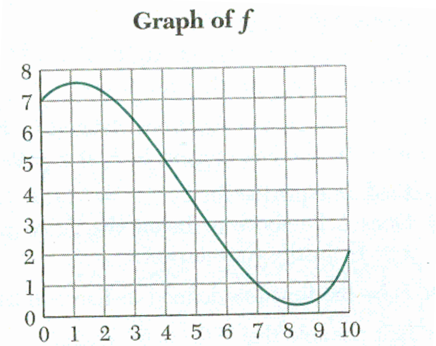
$f^{(3)}(x) = -6x^{-4}$

$f^{(4)}(x) = 24x^{-5}$

$f^{(n)}(x) = (-1)^n n!\, x^{-(n+1)}$

From HW: #33 and 75

F'=f



33. Let f be the function shown above, F be an antiderivative of f, and suppose that $F(0) = 10$. (The function f has a local maximum at $x = 1.14$, a point of inflection at $x = 4.67$, and a local minimum at $x = 8.19$.)

(a) What is the slope of F at $x = 4$? Justify your answer.

(b) Where is F increasing? Justify your answer.

(c) Where in the interval $[0, 5]$ does F achieve its largest value? Its smallest value? Justify your answers.

There are two different notations used for derivatives, the one we have been using is Newton's notation.
We also need to be familiar with Leibniz notation and will use it occasionally. Assume y=f(x), then we write:

$$\frac{dy}{dx} = f'(x)$$

Read this: derivative of y with respect to x

$$\left.\frac{dy}{dx}\right|_{x=a} = f'(a)$$

Read this: derivative of y with respect to x at x=a

NOTE: These are NOT fractions

If we differentiate with respect to x again:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) =$$

$$\frac{d^2y}{dx^2} = f''(x)$$

$$\frac{d^ny}{dx^n} = f^{(n)}(x)$$

What kind of functions can we now differentiate?
Anything that is a power function or a multiple of a power function or a linear combination of multiples of power functions. What set of functions does this include?

Polynomials: Definition of polynomial:
Whole number exponents

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

(has a derivative)

Theorem: Every polynomial is differentiable for all real numbers x. The derivative of every polynomial is another polynomial.

$$Q(x) = x^2 + x + 3$$

$$\frac{d(x^2+x+3)}{dx} = 2x+1$$

$$\frac{d^2(x^2+x+3)}{dx^2} = 2$$

$$\frac{d^3(x^2+x+3)}{dx^3} = 0$$

$$\frac{d^{1004}(x^2+x+3)}{dx^{1004}} = 0$$

A little more about Continuity

Recall that we learned:

There is Algebra with Limits

Suppose that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$

Need to do

where L and M are finite numbers (a is not necessarily finite).
Let k be any constant. Then the following limits all exist and are given by:

and we got all combinations (+ - x ÷) of limits to exist and do what we'd expect.

Also, remember the definition of continuity is

$$\lim_{x\to a} f(x) = f(a)$$

We then have: Algebra of Continuity:
If f and g are both continuous at x=a, then each of the functions f+g f-g fg are also continuous at x=a. If g(a)≠0, then f/g is also continuous at x=a.

We even get information about composition and continuity:

FACT: If f and g are functions such that $\lim_{x\to a} g(x) = b$ and f is continuous at x=b, then $\lim_{x\to a}(f\circ g)(x) = \lim_{x\to a} f\big(g(x)\big) = f(b)$

Theorem: Suppose that g is continuous at x=a and f is continuous at g(a). Then $f\circ g$ is continuous at x=a

Applications: Suppose we wanted to know:

$$\lim_{x\to 0}\ln\left(\frac{\sin x}{x}\right)$$

Need to do

$$\lim_{x\to 0}\ln(\cos x)$$