HW: Page 173 #1 - 5, 9, 10, 23, 24

# DO NOW:

28 33 p.161

Any questions from the HW? Write the problem # on the board.

Use all resources available to you to figure out the following:

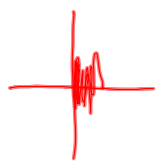
$$\lim_{x\to\infty} \sin x = \text{DNE, oscillates}$$

$$\lim_{x\to\infty} x\sin x = \text{DNE, oscillates wildly}$$

$$\lim_{x\to\infty} |x\sin x| = \text{DNE}$$

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

$$\lim_{x\to 0} \sin\left(\frac{1}{x}\right) = \text{DNE}$$



$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0$$



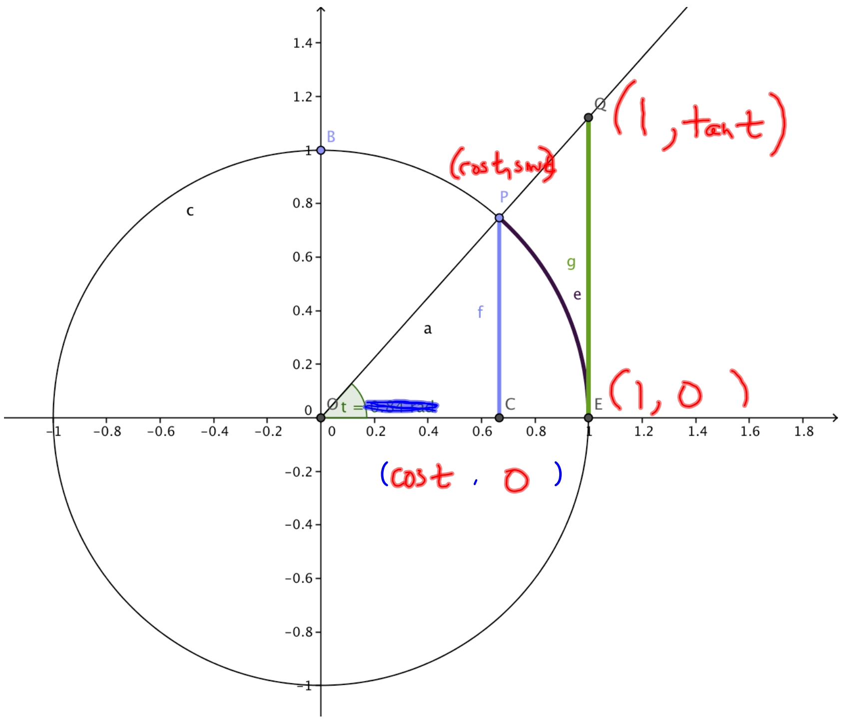


Squeeze Principle: Suppose that f(x)≤g(x)≤h(x) for all x near x=a.

If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$ then $\lim_{x\to a} g(x) = L$

We will use the Squeeze Principle to prove $\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$ which is crucial to our proof that the derivative of sin x is cos x

Our proof will use the diagram on the next page. Because sin t/ t is an even function, it will suffice to prove this theorem works as t approaches 0 from the right.




Write coordinates for C, E, P, Q



Squeeze Principle, lim sinx/x October 06, 2016

In our diagram, we have the following inequality, based on the properties of area:

Area(ΔOCP)≤Area(Sector OEP)≤Area (ΔOEQ)

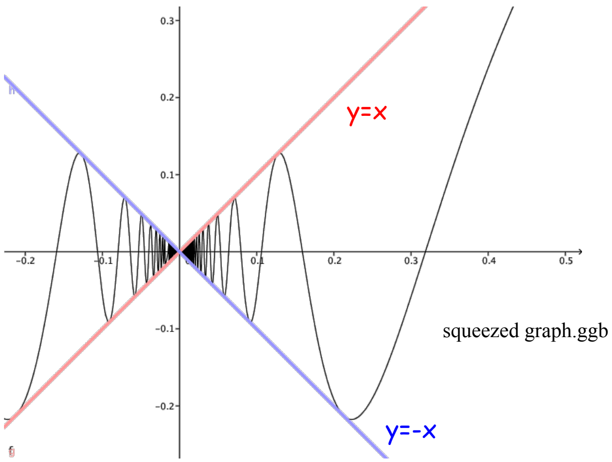
$$\frac{\sin t \cos t}{2} \le \frac{t}{2} \le \frac{\tan t}{2}$$

We then multiply through by $\frac{2}{\sin t} > 0$

to get $\cos t \le \frac{t}{\sin t} \le \frac{1}{\cos t}$

Since cos t →1 as t→0. we get our result by the Squeeze Principle.

$$\lim_{t \to 0^+} \frac{t}{\sin t} = 1 \qquad \frac{1}{2} r^2 \theta$$




We can use a similar argument to look at our old friend

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

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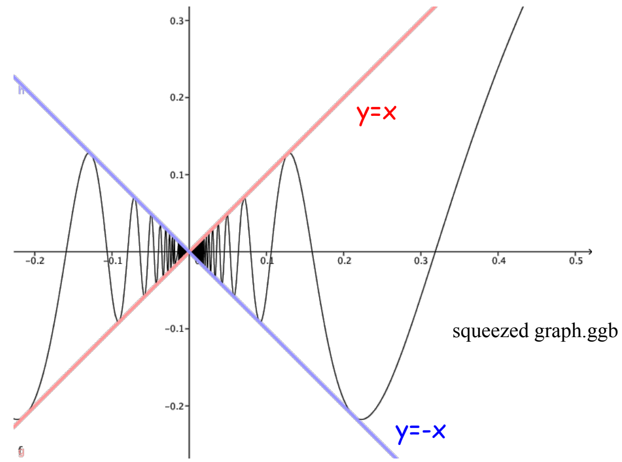


We can use a similar argument to look at our old friend

$$\lim_{x \to 0^+} -x \le \lim_{x \to 0^+} x \sin\left(\frac{1}{x}\right) \le \lim_{x \to 0^+} x$$

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## Page 176 #28

Let *f* be a function defined for all real numbers. Which of the following statements *must* be true about *f*? Which *might* be true? Which *must* be false? Justify your answers.
(SOMETIMES, ALWAYS, NEVER)

$$\lim_{x \to a} f(x) = f(a)$$

If $\lim_{x \to 0} \frac{f(x)}{x} = 2$ then f(0)=0.

If $\lim_{x \to 0} \frac{f(x)}{x} = 1$ then $\lim_{x \to 0} f(x) = 0$

If $\lim_{x \to 1^-} f(x) = 1$ and $\lim_{x \to 1^+} f(x) = 3$ then $\lim_{x \to 1} f(x) = 2$

If $\lim_{x \to 2} f(x) = 3$ then 3 is in the range of *f*.

If $\lim_{x \to 0} \frac{f(x) - f(0)}{x} = 3$ then $f'(0)=3$