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DO NOW:

1) If you have questions about any of the HW problems, please put the problem number on the board.

2)

Let $f(x)=\begin{cases} bx^2+1 & \text{if } x<-2 \\ x & \text{if } x\geq -2 \end{cases}$ $\quad f(-2) = -2$

What value of b makes f continuous at x= -2?

$\lim_{x\to -2^-}$

$f(-2) = -2$

$b(-2)^2+1 = -2$

$4b = -3$

$b = -3/4$

We need to learn about "tricky" limits in order to study derivatives. Consider the following:

Let f(x)=x^2 What is f'(3)?

$$f'(3) = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3} \frac{x^2-9}{x-3}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$$

$$= \lim_{x\to 3} x+3$$

$$= 6$$

3) For f(x)=sin (x), how do we use the limit definition to find f'(0)?

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\sin x}{x}$$

Let's look at that limit on our calculator and see what we get.

First time through, make sure your calculator is in radian mode.

Looks like one.

Then, see what happens when the calculator is in degree mode.

CAVEAT $.01745 \approx \frac{\pi}{180}$

Calculator always in radian mode for Calculus!!

First make an educated guess and then use your calculator, to try to figure out what the following limits are (if they exist):

a) $\lim_{x\to\infty} \sqrt{x^2+1} - x = 0$

b) $\lim_{x\to\infty} \sqrt{x^2+x} - x = 0.5$

My suggestion is to make a table, where the independent variable is set to ask. Try x values like 10^2, 10^3, 10^4, 10^6, etc.

What do we mean when we write

Horizontal asymptote, $y=b$

$\lim_{x\to\infty} f(x) = b$ means as x increases without bound, then $f(x)$ gets closer to b

from both sides

$\lim_{x\to a} f(x) = \infty$ as x approaches a, then $f(x)$ increases without bound

Vertical asymptote, $x=a$

How are these kinds of limits related to vertical and horizontal asymptotes on the graph of f(x)?

There is Algebra with Limits

Suppose that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$

where L and M are finite numbers (a is not necessarily finite). Let k be any constant. Then the following limits all exist and are given by:

1) $\lim_{x\to a} kf(x) = kL$

2) $\lim_{x\to a} f(x)\cdot g(x) = LM$

3) $\lim_{x\to a} [f(x)+g(x)]$ $L+M$

4) $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ $\dfrac{L}{M}$ $M \neq 0$

Because ∞ is not a number, we need to be careful to remember that the rules of arithmetic do NOT apply when we have ∞
We will use shorthand sometimes and behave as if we have a new arithmetic for ∞, but we need to remember that it's not one of our friends that we met back in elementary school.

Some new arithmetic that DOES WORK:

1/∞=0 is okay to say if we mean it's short for $\lim_{x\to\infty}\frac{1}{x}=0$

3*∞=∞ is okay. If a quantity increases without bound, then so does three times that quantity.

∞+∞=∞ also works. The sum of two quantities that increase without bound also increases without bound.

The following are INCORRECT:

1/0→∞ NO! if you're looking at 1/x as x goes to 0 the hands definitely don't clap; one goes to +∞ and the other to -∞
(this is really hard on your shoulders!)

∞-∞=0 If you subtract two quantities that increase without bound, you can get any result you like;

$$\lim_{x\to\infty}\left((x+29)-x\right)=29$$

∞/∞=1 If you divide two quantities that increase without bound, you can get any result you like; (where have I heard that before?)

$$\lim_{x\to\infty}\frac{3x^2+x}{x^2}=3$$

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