

HW : Page 161 # 39 - 47 (odd)



If the work is worthwhile, then whether we can complete it or not, it's worth making the attempt. That's why courage is important.

Do Now:

Do problems #2 - 27 about the graphs of f and g.

This is based on material you did last year, see what you remember.

Front of Room

Windows

|                      |                  |                    |                    |                  |                 |                   |
|----------------------|------------------|--------------------|--------------------|------------------|-----------------|-------------------|
| Kolacznyk<br>Carolyn | Jasinskis<br>Zoe | Gunnig<br>Maren    | Bastepo<br>Isisou  | Chen<br>Winnie   | Waltz<br>Ava    |                   |
| Chin<br>Alex         | Gavish<br>Itai   | Reilly<br>Harrison | Bennett<br>Max     | Razis<br>Costa   | Zhang<br>Nick   | Gilman<br>Gersten |
| Garish<br>Einat      | Gilman<br>Noah   | Kahale<br>Tamara   | Cohen<br>Sophie    | Lallier<br>Julia | Shen<br>Sophie  |                   |
| Grossman<br>Gabby    | Perlo<br>Devin   | Scott<br>Nick      | Klein<br>Nathaniel | Liao<br>Iris     | Reilly<br>Liana |                   |

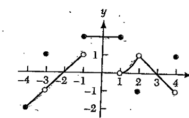
Page 160 - Limits from Graphs

BASICS

- Let  $f(x) = \begin{cases} 1, & \text{if } x \text{ is an integer} \\ 2, & \text{if } x \text{ is not an integer} \end{cases}$ .
  - Draw a graph of  $f$  over the interval  $[0, 5]$ .
  - Evaluate  $\lim_{x \rightarrow 1} f(x)$ .
  - Evaluate  $\lim_{x \rightarrow 0.5} f(x)$ .
  - For which values of  $a$  does  $\lim_{x \rightarrow a} f(x)$  exist? Why?

Let  $f$  be the function whose graph is shown below. Using the graph, evaluate each of the limits in Exercises 2-15 or explain why the limit does not exist. [NOTE:  $f(1) = 2$ .]

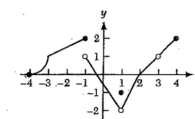
Graph of f



- $\lim_{x \rightarrow 2} f(x)$  2
  - $\lim_{x \rightarrow -3} f(x)$  dne
  - $\lim_{x \rightarrow -1} f(x)$
  - $\lim_{x \rightarrow 1} f(x)$
  - $\lim_{x \rightarrow 2} f(x)$
  - $\lim_{x \rightarrow 2^+} f(x)$  1
  - $\lim_{x \rightarrow 2} f(x)$
  - $\lim_{x \rightarrow 2} f(x)$  0
  - $\lim_{x \rightarrow 2} f(x)$
  - $\lim_{x \rightarrow 1^+} f(x)$
  - $\lim_{x \rightarrow 2} f(x)$  -2
  - $\lim_{x \rightarrow -1^+} f'(x)$
  - $\lim_{x \rightarrow -1^-} f'(x)$
  - $\lim_{x \rightarrow -1} f'(x)$  1
  - $\lim_{x \rightarrow -3} f'(x)$
16. Over which intervals is  $f$  continuous?

Let  $g$  be the function whose graph is shown below. Using the graph, evaluate each of the limits in Exercises 17-26 or explain why the limit does not exist.

Graph of g



- $\lim_{x \rightarrow 4^+} g(x)$
- $\lim_{x \rightarrow -3} g(x)$  1
- $\lim_{x \rightarrow 2} g(x)$
- $\lim_{x \rightarrow -1} g(x)$  1
- $\lim_{x \rightarrow 1} g(x)$
- $\lim_{x \rightarrow -1} g(x)$
- $\lim_{x \rightarrow 4^+} g(x)$  0.5
- $\lim_{x \rightarrow 0} g(x)$
- $\lim_{x \rightarrow 2} g(x)$  0
- $\lim_{x \rightarrow 1} g(x)$
- $\lim_{x \rightarrow 2} g(x)$
- $\lim_{x \rightarrow 1} g(x)$  2
- Over which intervals is  $g$  continuous?

What are our two limit definitions of the derivative?

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

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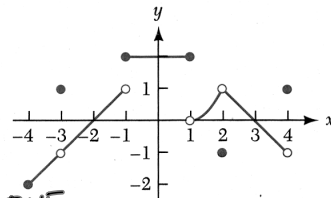
Use the graph of  $f$  and  $g$ , and the definition of the derivative (as appropriate) to evaluate each limit. If the limit does not exist, explain why.

51.  $\lim_{x \rightarrow 0} \frac{f(x) - 2}{x - 0} = f'(0) = 0$

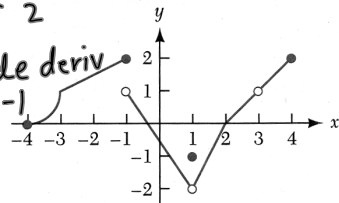
52.  $\lim_{x \rightarrow 3} \frac{f(x)}{x - 3} = f'(3) = -1$

Wrong answer in 53. packet  
 53.  $\lim_{x \rightarrow 1} \frac{f(x)}{x + 2} = \frac{0}{3} = 0$  NOT A DERIVATIVE

Graph of  $f$



Graph of  $g$



54.  $\lim_{h \rightarrow 0} \frac{g(-2+h) - 3/2}{h} = g'(-2) = 1/2$

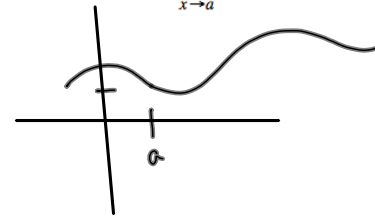
55.  $\lim_{h \rightarrow 0^-} \frac{g(-1+h) - 2}{h} = \text{left-side deriv of } g \text{ at } -1 = 1/2$

56.  $\lim_{h \rightarrow 0} \frac{g(2+h)}{h} = g'(2)$ , does not exist

What is our intuitive definition of limits?

What do we mean when we write

$$\lim_{x \rightarrow a} f(x) = L$$



For  $x$  values closer to  $a$ , then the  $y$ -values are closer to  $L$

## CONTINUITY

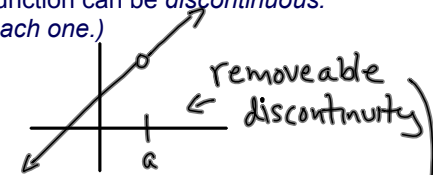
What is our definition of continuity?

We say  $f(x)$  is continuous at  $x=a$  if

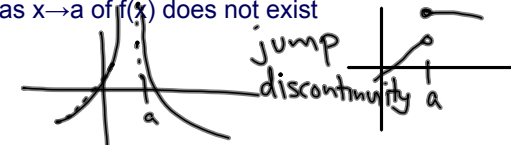
- 1)  $\lim_{x \rightarrow a} f(x)$  exist
- 2)  $f(a)$  exists
- 3)  $\lim_{x \rightarrow a} f(x) = f(a)$

So there are three ways a function can be *discontinuous*:  
(Generate an example for each one.)

- 1)  $f(a)$  is not defined



- 2) the limit as  $x \rightarrow a$  of  $f(x)$  does not exist



- 3) the limit as  $x \rightarrow a$  of  $f(x)$  exists but does not equal  $f(a)$



Example:

$$\text{Let } f(x) = \begin{cases} bx^2 + 1 & \text{if } x < -2 \\ x & \text{if } x \geq -2 \end{cases}$$

What value of  $b$  makes  $f$  continuous at  $x = -2$ ?