What big questions do you still have?

Look at HW w/ your group.

Recognizing a limit as a derivative

Recall our definition: $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Look at the expression below:

$$\lim_{x \to 1} \frac{e^x - e}{x - 1}$$

Can you recognize this as a derivative?

What is f(x)? e^x

What is a? 1

504: AP Calculus BC
2.5: Definition of the Derivative

Name________________________
Date________________________

1) What does the following represent for $g(x)$?

$$\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = g'(x)$$

2) Given $f(x)=x^3$, what does the following limit represent?

$$\lim_{h\to 0}\frac{(2+h)^3-8}{h} = f'(2)$$

$f(x)=x^2$

3) If $y=x^2$, what does the following limit represent for the function?

$$\lim_{h\to 0}\frac{(3+h)^2-3^2}{h}=6=f'(3)$$

4) The following limit represents the derivative for a particular function at a particular value of x.
What is $f(x)$? What is the value of a?

$$\lim_{h\to 0}\frac{3\sin\left(\frac{\pi}{7}+h\right)-3\sin\left(\frac{\pi}{7}\right)}{h}=f'(a)$$

$a=\frac{\pi}{7}$

$f(x)=3\sin x$

5) What does this limit represent?

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$$

504: AP Calculus BC
2.5: Definition of the Derivative

Name________________________
Date________________________

1) What does the following represent for $g(x)$?

$$\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = g'(x) \Rightarrow \text{AROC on } [x, x+h]$$

2) Given $f(x)=x^3$, what does the following limit represent?

$$\lim_{h\to 0}\frac{(2+h)^3-8}{h} = f'(2)$$

3) If $y=x^2$, what does the following limit represent for the function?

The slope at $x=3$

$$\left.\frac{dy}{dx}\right|_{x=3}=y'(3) \qquad \lim_{h\to 0}\frac{(3+h)^2-3^2}{h}=6 \quad y'(3)$$

4) The following limit represents the derivative for a particular function at a particular value of x.
What is $f(x)$? What is the value of a?

$$\boxed{\frac{d^2y}{dx^2}}=f''(x)$$

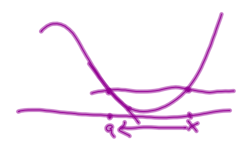
$$\frac{d\left(dy/dx\right)}{dx}=f''(x)$$

$$\lim_{h\to 0}\frac{3\sin\left(\frac{\pi}{7}+h\right)-3\sin\left(\frac{\pi}{7}\right)}{h}=f'(a)=\left.\frac{d(3\sin x)}{dx}\right|_{x=\frac{\pi}{7}}$$

$f(x)=3\sin(x)$

$a=\frac{\pi}{7}$

5) What does this limit represent?



$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$$

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=y'$$

GREATEST HITS for UNIT 1b

Students will be able to:

— everything from before . . . AND . . .

- know how to use properties of the first and second derivative of a function, f, to identify characteristics of f

- use the first derivative test to classify local extreme values and JUSTIFY with appropriate structure of language and use of notation

- Use the derivative to write equations of tangent lines

and Points of Inflection

- justify the concavity of a function given information about the first and second derivatives of the function

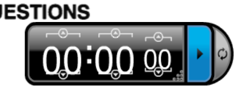
- use the second derivative test to classify local extrema and JUSTIFY with appropriate structure of language and use of notation

— Finding / Justifying Absolute Max/Min on a closed interval.

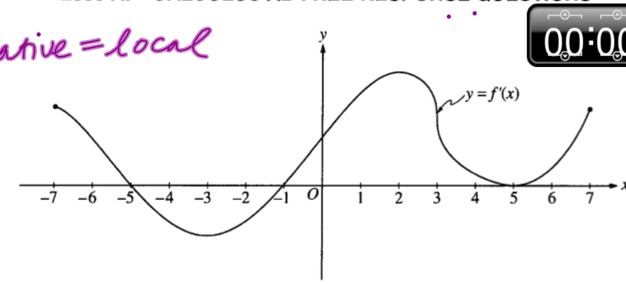
- use the properties of the derivative to distinguish between the graphs of f, f' and f'' (think "curve sketching")

- use limits to connect average rate of change to instantaneous rate of change/derivative (the limit definition of the derivative and its various forms)

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



relative = local



3. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

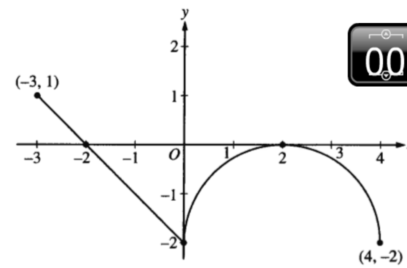
   (a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

   (b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

   (c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

   (d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

f has a local minimum at (a) $x=-1$, because $f'(x)$ changes from negative to positive at $x=-1$	2: 1: answer; 1: justification
f has a local max at (b) $x=-5$, because $f'(x)$ changes from positive to negative at $x=-5$	2: 1: answer; 1: justification
(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7,-3)$, $(2,3)$, and $(3,5)$	2: 1: $(-7,-3)$; 1: $(2,3)\cup(3,5)$
(d) $x=7$ The absolute maximum must occur at $x=-5$ or at an endpoint. $f(-5) > f(-7)$ because f is increasing on $(-7,-5)$ The graph of f' shows that the magnitude of the negative change in f from $\underline{x=-5 \text{ to } x=-1}$ is smaller than the positive change in f from $\underline{x=-1 \text{ to } x=7}$. Therefore the net change in f is positive from $x=-5$ to $x=7$, and $f(7) > f(-5)$. So $f(7)$ is the absolute maximum.	3: 1: answer; 1: identifies $x=-5$ and $x=7$ as candidates — or — indicates that the graph of f increases, decreases, then increases; 1: justifies $f(7) > f(-5)$



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

(a) The function f is increasing on $[-3,-2]$ since $f' \geq 0$ for $\cancel{-3 \le x < -2.}$ those values.	$2: \begin{cases} 1: \text{interval} \\ 1: \text{reason} \end{cases}$
there are points of infection @ (b) $x = 0$ and $x = 2$ because f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$	$2: \begin{cases} 1: x = 0 \text{ and } x = 2 \text{ only} \\ 1: \text{justification} \end{cases}$
(c) $f'(0) = -2$ $f(0) = 3$ Tangent line is $y = -2x + 3$.	1 : equation
(d) $f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$ $f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$ $f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$ $f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$	4 : 1 : $\pm\left(\frac{1}{2} - 2\right)$ (difference of areas of triangles) 1 : answer for $f(-3)$ using FTC 1 : $\pm\left(8 - \frac{1}{2}(2)^2\pi\right)$ (area of rectangle − area of semicircle) 1 : answer for $f(4)$ using FTC