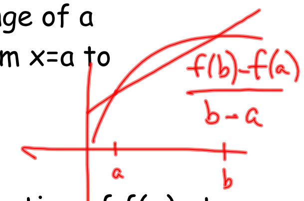
HW: Packet: What does f' tell us about f?

Do Now: Spend a few minutes comparing answers to the HW. We will go over this in class tomorrow.

Comparing Average and Instantaneous Rates of Change

FACT 1: The average rate of change of a function f(x) over the interval from x=a to x=b is given by

$$\frac{f(b)-f(a)}{b-a}$$



FACT 2: We have defined the derivative of f(x) at x=a, f'(a), to be the instantaneous rate of change of f(x) at x=a

How are these two facts related?

504: BC Calculus
2.5 Activity (Definition of the Derivative)

Name:______________________
Date:______________________

1) Identify each of the following difference quotients:

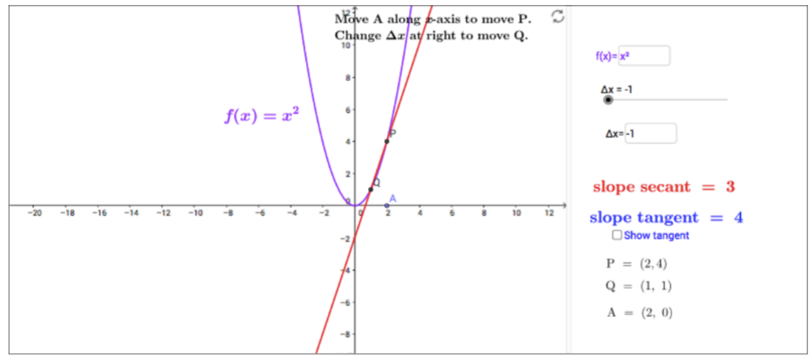
$[x, x+h]$	$[x-h, x]$	$[x-h, x+h]$
$\frac{f(x+h)-f(x)}{h}$	$\frac{f(x)-f(x-h)}{h}$	$\frac{f(x+h)-f(x-h)}{2h}$
Right Diff Quotient	Left Diff Quotient	Symmetric Difference Quotient
$Y_2 = \frac{Y_1(x+H)-Y_1(x)}{H}$	$Y_3 =$	$Y_4 =$

2) What do these difference quotients represent?

Enter these difference quotients into your graphing calculators while we are discussing them!

Secant to Tangent Line

Draws secant line between two points on function. Secant line moves closer to tangent line as second point is moved closer to first. Use slider to move second point.



Now...use this information to complete the activity...(10 minutes)

3) Enter each of the following difference quotients into your calculator, letting $f(x)=Y_1$.
4) Set up your table so that Tblstart=1 and ΔTbl =1.
5) Go to the home screen and let H=1 ($1 \to H$)
6) Examine the pattern you see in the table and express Y_2, Y_3, Y_4 as functions of x.

8) What is happening to H in each of the subsequent tables? $H \to 0$
9) What is happening to the equations for Y_2, Y_3, Y_4 in each of the subsequent tables? Identify the pattern.

$Y_2 = 2x+H \quad Y_3 = 2x-H \quad Y_4 = 2x$

10) What BIG idea is this idea trying to get across?

$Y_2 \to Y_3 \Rightarrow Y_4$

$\lim_{H\to 0} 2x+H \to 2x$

$\lim_{H\to 0} 2x \to 2x$

$\lim_{H\to 0} 2x-H \to 2x$

Additional Concluding Thoughts:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = 2x$$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = f'(2) = 4$$

$$\boxed{f(x)=x^2}$$

$$\lim_{h\to 0} \frac{(2+h)^2-4}{h} = \lim_{h\to 0} \frac{\cancel{4}+4h+h^2-\cancel{4}}{h} = \lim_{h\to 0} \frac{4h+h^2}{h} = \lim_{h\to 0} 4+h \to 4$$

$$\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$$

$$\lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h \to \boxed{f'(x)=2x}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$f(x)=x^2$

$f'(3)$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$\lim_{h\to 0} \frac{(3+h)^2-9}{h}$$

$$\lim_{h\to 0} \frac{\cancel{9}+6h+h^2-\cancel{9}}{h}$$

$$\lim_{h\to 0} 6+h$$

$$\boxed{f'(3)=6}$$

$H \rightarrow 0$

Y_2, Y_3, Y_4 closer to each other

BIG IDEA: "Any Difference Quotient"

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

For example: For the function $f(x) = x^2$

The slope of a secant that is "h" away from 2 is given by

$$\frac{\Delta f}{\Delta x} = \frac{f(2+h)-f(2)}{(2+h)-2} = \frac{f(2+h)-f(2)}{h}$$

We will say that f'(2) is the limit of these slopes as h→0

$f'(2)$

slope of secant to x squared.ggb

$$\frac{f(2+h)-f(2)}{h} = \frac{4+4h+h^2-4}{h} =$$

http://www.slu.edu/classes/maymk/GeoGebra/SecantToTangent.html

$$\underbrace{\frac{h(4+h)}{h}}_{g(h)} \overset{?}{\neq} \underbrace{4+h}_{j(h)}$$

$$\lim_{h\to 0} \frac{h(4+h)}{h} \overset{?}{=} \lim_{h\to 0} 4+h$$

Yes!

DEFINITIONS

Difference quotient= $\frac{f(b)-f(a)}{b-a}$

$\frac{f(a+h)-f(a)}{h}$ Slopes of secants

If f is defined near AND at x=a, then the derivative of f at x=a is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

NOTE: The units for f'(x) are the units of the difference quotient

Links for you to try...

http://www.dougkuhlmann.webs.com/

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_try_to_graph.html

http://www.math.uri.edu/~bkaskosz/flashmo/derplot/