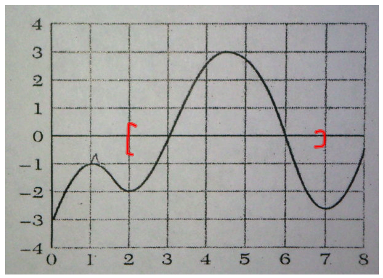
HW: Page 135 #1, 4 , 6, 20, Justifications Worksheet

Checkin on Friday, Sept. 30, 2016

Unit 1bTest: Friday, Sept. 30, 2016

DO NOW: Problem #26, page 129



The graph of the derivative of a function *f* is shown to the right. Use the graph of *f'* to answer the following questions about *f*. *[NOTE: The graph of f is not shown.]*

Whenever $x_1 < x_2$ then $f(x_1) < f(x_2)$

a) On what intervals is *f* increasing? Decreasing?

[3,6] [0,3] [6,8]

b) Where does *f* have a stationary point?

x=3, 6 f'(x)=0 at those points

c) Where does *f* have a local maximum? Local minimum?

x=3 is local min, because f'(x) goes neg to pos. x=6 local max f'(x) goes pos to neg

d) On which intervals is *f* concave up? Concave down?

When f'(x) is increasing [0,1] [2,4.5] [7,8] down f'(x) dec [1,2] [4.5,7]

e) Where does *f* have a point of inflection?

when f'(x) changes direction x=1, 4.5, 7

f) Where does *f* achieve its maximum value on the interval [0,2]? Its minimum value?

f'(x)<0 on [0,2] so f(x) dec on [0,2], so f(0) is max value f(2) is min value

g) Where does *f* achieve its maximum value on the interval [3, 6]? Its minimum value?

f'(x)>0 on [3,6] so f(x) inc on [3,6] max value is f(6) min value is f(3)

h) Assume that *f*(0)=0. Sketch a graph of *f*. (Your graph need only have the right general shape.)

i) How does your answer to part h) change if *f*(0)=5?

More definitions:

f(x) is concave up on an interval *I* if f ' is increasing on *I*.

holds water

f(x) is concave down on an interval *I* if f ' is decreasing on *I*.

spills water

inflection point - point at which the direction of concavity changes (which is when direction of f' changes)

HIGHER DERIVATIVES

If f(x) is a function, then its derivative f'(x) is a function. So, we can take the derivative of f'(x) and get the second derivative of f(x), which we notate as f"(x). The derivative of f"(x) is written f'''(x). After that, we switch to the notation $f^{(4)}(x)$ for higher derivatives.

What does the second derivative of a function tell us about the function?
and its graph

Remember what we said the derivative tells us about f(x)
If f'(x)>0 on an interval then f(x) is increasing on interval

If f'(x)<0 on an interval then f(x) is decreasing on interval

We also defined concavity:
If f'(x) is increasing on an interval, then f is concave up on the interval.
If f'(x) is decreasing on an interval, then f is Concave down on the interval.

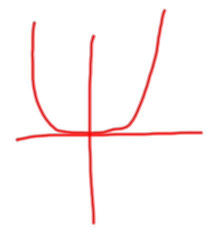
Putting these two together for f"(x) we get:
If f"(x)>0 on an interval, then f(x) is Concave up on interval.
If f"(x)<0 on an interval, then f(x) is concave down on interval.

Where do we find inflection points?

Concavity can only change where the second derivative changes sign, that is where f"(x)=0 or f"(x) is undefined.
REMEMBER: The converse if NOT TRUE! (This is the same idea as for the first derivative; not every stationary point is a local max or local min.

What is our counterexample?

$f(x)=x^4$



We can also use the second derivative to decide whether a stationary point is a local max or local min.

If a function is concave up at a stationary point, then the point is local min

If a function is concave down at a stationary point, then the point is local max

This gives us the second derivative test:

Suppose that f'(a)=0. There are three possibilities:

If f"(a)>0, then f(x) has a **local min** at x=a.

If f"(a)<0, then f(x) has a **local max** at x=a.

If f"(a)=0, then NO CONCLUSION CAN BE DRAWN.

Higher Derivatives and Second Derivative Test September 22, 2016

Log into the Chromebook, and go to my website, ghitelman.pbworks.com and follow the link on the assignment page to Marc Renault's Calculus Applets

Do #10, 20, 12, 14, 15