Agenda (Day 5): **The Racetrack Principle (Speed Limit Law)**

SWBAT use the derivative to predict bounds for the value of a function on a given interval given a point and restrictions for the rate of change of the function on the interval.

HW: Page 107 #21, 23, 29 (part (c) is a challenge), 32

1) Homework Check and Q/A (5 -10 min)

2) Racetrack (quick video) and then in your groups sketch a graph to represent parts I and IV. (5-7 min)

3) A few examples using the Racetrack Principle (10 - 15 min)

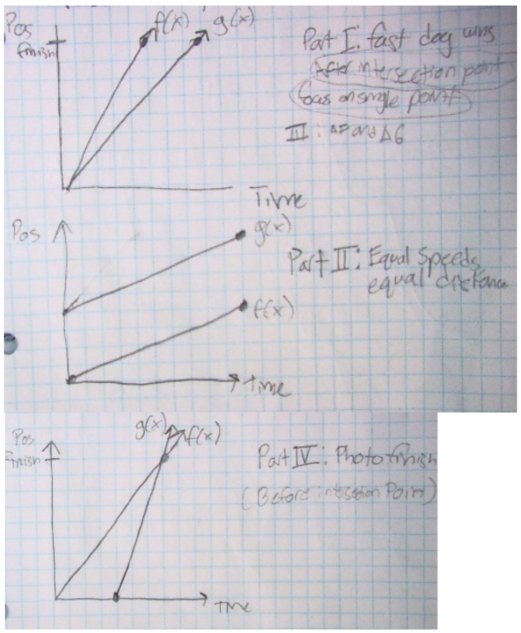
4) Review Card Expectations

http://www.youtube.com/watch?v=1rnhEK_z6TY

a) In your groups sketch a graph to represent parts I and IV.

b) What distinguishes the two?

Graphical Representation of the Racetrack Principle



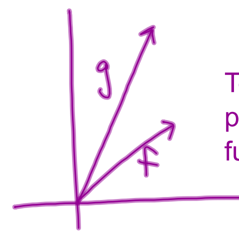
http://www.youtube.com/watch?v=1rnhEK_z6TY

RACETRACK PRINICIPLE

PART I: Fast dogs win.

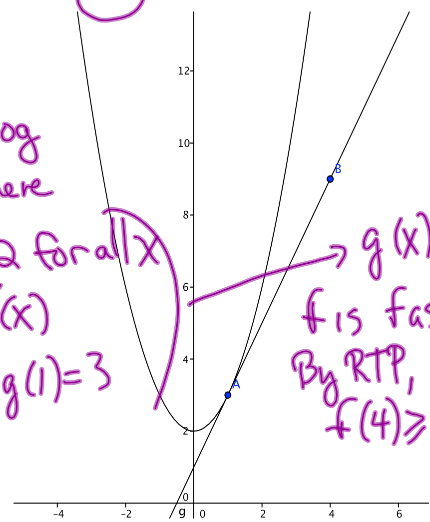
(INFORMAL) If dogs *f* and *g* start together and dog *g* always runs faster than dog *f*, then dog *g* always leads and therefore wins the race.

(FORMAL) Let *f* and *g* be functions defined for all x in $[a,b]$, and suppose that $f(a)=g(a)$. If $f'(x)\le g'(x)$ for all x in $[a,b]$, then $f(x)\le g(x)$ for all x in $[a,b]$.



To use when we want to look at a point after the place where the two functions intersect (larger x value)

How can we use this to prove properties of functions?
Example: (#22, page 107)
Suppose that f(1)=3 and f'(x)≥2 for all x in [0,5]. Show that f(4)≥9.



Pick a slower dog $g(x)$, where $g'(x)=2$ for all x → $g(x)=2(x-1)+3$

$f'(x)\geq g'(x)$

$f(1)=g(1)=3$

f is fast dog,
By RTP,
$f(4)\geq g(4)=2(4-1)+3$
$=9$

PART IV: Photo finish

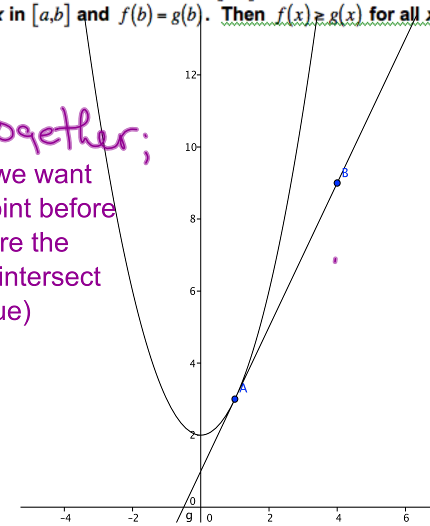
(INFORMAL) If dogs *f* and *g* finish together, then the faster dog cannot have been ahead at any time.

PART IV: Photo finish

Let *f* and *g* be functions defined for all *x* in $[a,b]$, and suppose that $f'(x)\leq g'(x)$ for all *x* in $[a,b]$ and $f(b)=g(b)$. Then $f(x)\geq g(x)$ for all *x* in $[a,b]$.

end together;

To use when we want to look at a point before the place where the two functions intersect (smaller x value)



Example: (#22, page 107)

Suppose that f(1)=3 and f'(x)≥2 for all x in [0,5]. Show that f(0)≤1.

Note: Same conditions as previous problem, so our slow dog is the same

Pick $g(x)$ so that $g'(x)=2$

$g(1)=3$

$f'(x) \geq g'(x)$ $\quad g(x)=2(x-1)+3$

For finding a bound on $f(0)$, we use Photo Finish which says the faster dog is always behind

So $f(0) \leq g(0) = 2(0-1)+3 = 1$

faster dog slower dog

21. Suppose that $f(1) = -2$ and $f'(x) \leq 3$ for all x in $[-10, 10]$.
   (a) Show that $f(2) \leq 1$. $[1,2]$ Part 1 (after pt. of intersection)
   (b) Show that $f(5) \leq 10$.
   (c) Show that $f(0) \geq -5$.
   (d) Is $f(9) < 31$? Justify your answer.
   (e) Is $f(-5) > -23$? Justify your answer.
   (f) Could $f(4) = 8$? Justify your answer.
   (g) Could $f(-7) = -25$? Justify your answer.
   (h) Could $f(8) = -25$? Justify your answer.
   (i) Let $g(x) = 4x + 3$. Compare the numbers $f(7)$ and $g(7)$.

(a) Let $g' = 3$ and $g(1) = f(1) = -2$

and $g(x) = 3(x-1) - 2$

Since $f' \leq g'$ on $[1,2]$

and $g(1) = f(1) = -2$

then $f(2) \leq g(2) = 1$

$f(2) \leq 1$ QED

Let $g' = 3$ and $g(1) = f(1) = -2$

and $g(x) = 3(x-1) - 2$

Since $f' \leq g'$ on $[0,1]$

and $g(1) = f(1) = -2$

~~$g(0) < f(0)$~~

$f(0) \geq g(0) = -5$

$f(0) \geq -5$ QED

part g of #21

Suppose that $f(1) = -2$ and $f'(x) \le 3$ for all x in $[-10, 10]$

could $f(-7) = -25$; Justify your answer

Let $[-7, 1]$

$g'(x) = 3$ and $\boxed{f(1) = g(1) = -2}$, so $\Rightarrow g(x) = 3(x-1) - 2$

(since $f'(x) \le g'(x)$ on $\boxed{[-7, 1]}$ $\quad g(-7) = -26$

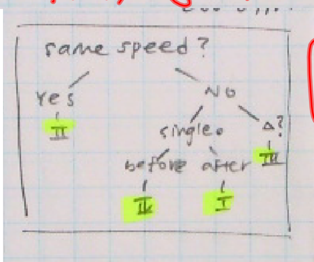
and + $g(1) = f(1) = -2$

then $g(-7) \le f(-7)$ ✓ YES IT CAN!

$-26 \le -25$

$f(-7) \ge g(-7) = -26$

$f(-7) \ge -26$



$\boxed{f(-7) \text{ could be } -25.}$