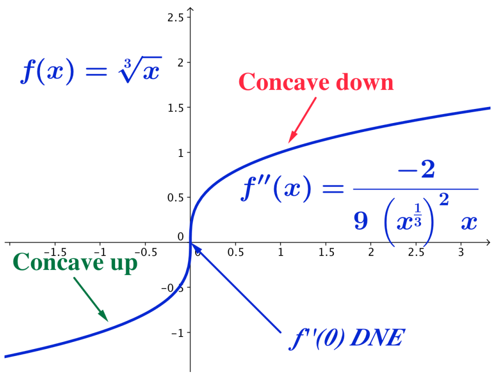
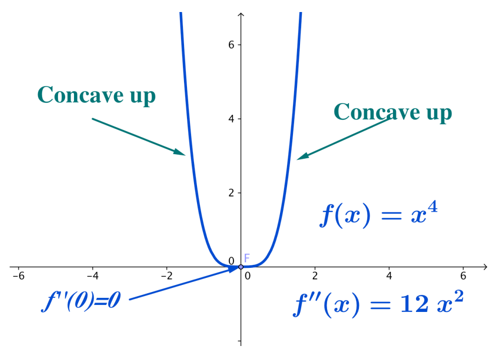
Top Ten Student Errors (with corrections and explications)

1. $f''(x)=0 \Leftrightarrow (x, f(x))$ is a point of inflection. Must show that f'' changes signs, not just that it is 0. Also f'' could be undefined at a point of inflection.
Correct statement: If $(c, f(c))$ is a point of inflection AND $f''(c)$ exists, then $f''(c)=0$. Example 2 below shows the converse is false.

Example1: Let $f(x)=x^{1/3}$, then $f''(x)=-\frac{2}{9}x^{-5/3}$ which does not exist at 0. However, the function is continuous at 0, has a vertical tangent there and changes concavity at 0. Note that $f''(x)>0$ if $x<0$ and $f''(x)<0$ if $x>0$. So $(0,0)$ is a point of inflection even though $f''(0)$ *DNE*





Example 2: Consider $f(x)=x^4$. Then $f''(x)=12x^2$ and $f''(0)=0$ but (0,0) is not a point of inflection since f'' does not change sign there.

Bottom line: You must show the second derivative changes sign, not just that is 0.

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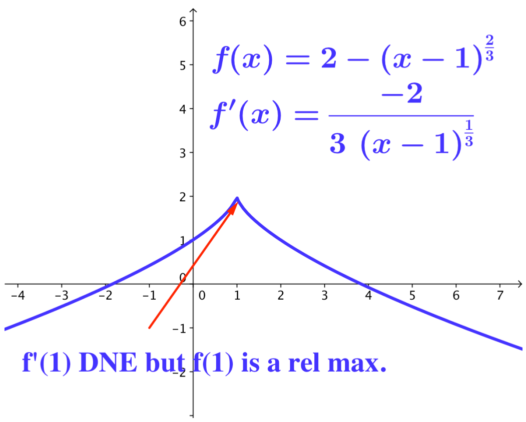
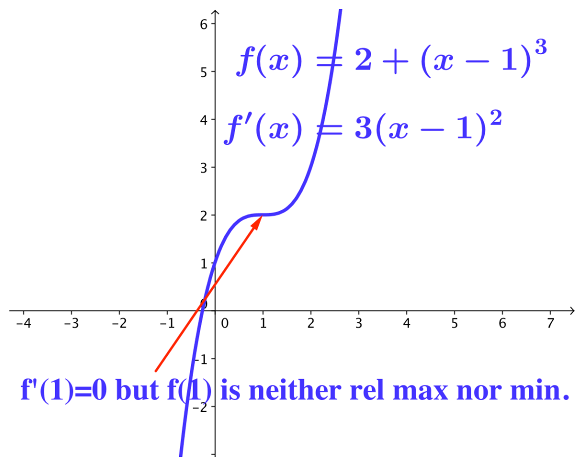
2. $f(x)$ is a maximum (minimum) $\Leftrightarrow f'(x)=0$. Similar to above. Must show that f' changes from positive to negative for max and v.v. for min. Also f' could be undefined at max/min.
Correct statement: If $(c, f(c))$ is a relative extremum AND $f'(c)$ exists, then $f'(c)=0$.

   Two examples (similar to the examples in 1)





   Bottom line: You must show f' changes sign at $x=c$.

3. Average rate of change of f on [a, b] is $\dfrac{f'(a)+f'(b)}{2}$. Should be $\dfrac{f(b)-f(a)}{b-a}$

4. Volume by washers is $\int_a^b \pi (R-r)^2\,dx$. Should be $\int_a^b \pi (R^2 - r^2)\,dx$

5. Separable differential equations can be solved without separating the variables. Make sure you do not have x and y on the same side (this includes dx and dy). I.E., all x's with the dx and all y's with the dy.

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6. Omitting the constant of integration, especially in initial value problems. Be sure to add the constant to one side as soon as you find the antiderivatives and before you start modifying the equation, say, by squaring both sides or taking the ln of both sides.

   Example of incorrectly adding C: Solve $\frac{dP}{dt} = 2P$ with P=5 when t=0.

   Separate variables (correctly): $\frac{1}{P}dP = 2dt \Rightarrow \ln|P| = 2t$ or $P = e^{2t}$ +C ***

   Initial condition implies C=4 so $P = e^{2t} + 4$. The mistake was adding C after taking the exponential of both sides. It should go like this: $\frac{1}{P}dP = 2dt \Rightarrow \ln|P| = 2t + C$, that is add the C as soon as you find the antiderivative. Then you get $P = e^{2t+C} = Ae^{2t}$
   Initial condition implies A=5.

7. Graders will assume the correct antecedents for all pronouns used in justifications. She doesn't know what 'it' refers to. Example: "When it is zero, the function has a maximum."

8. If the correct answer came from your calculator, the grader will assume your setup was correct. These are called "bald" answers. No justification so you will not get full credit-- or any credit sometimes.

9. Universal logarithmic antidifferentiation: $\int \frac{1}{f(x)}dx \; = \; \ln|f(x)| + C$.

   This is correct: $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$ Note that you must have $\frac{f'(x)}{f(x)}$ in the integrand. Without the f' you must use some other integration technique.

10. $\frac{d}{dx}f(y) = f'(y)$ and other Chain Rule errors. Should be $\frac{d}{dx}f(y) = f'(y)\frac{dy}{dx}$

    Example: If $\frac{dy}{dx} = x^2y^3$, find $\frac{d^2y}{dx^2}$. INCORRECT: $\frac{d^2y}{dx^2} = 2xy^3 + 3x^2y^2$. The student forgot that the y^3 bit fit the pattern in 10.

    CORRECT: $\frac{d^2y}{dx^2} = 2xy^3 + 3x^2y^2\frac{dy}{dx} = 2xy^3 + 3x^2y^2(x^2y^3) = 2xy^3 + 3x^4y^5$

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Disclaimer by Dan Kennedy
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